Mathematics
Arithmetic Progression and Geometric Progression
(Core)

CENTRAL BOARD OF SECONDARY EDUCATION
Shiksha Kendra, 2, Community Centre, Preet Vihar, Delhi-110 092 India
Mathematics
Arithmetic Progression and Geometric Progression
(Core)

CLASS X
UNIT-5
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The Curriculum initiated by Central Board of Secondary Education -International (CBSE-i) is a progressive step in making the educational content and methodology more sensitive and responsive to the global needs. It signifies the emergence of a fresh thought process in imparting a curriculum which would restore the independence of the learner to pursue the learning process in harmony with the existing personal, social and cultural ethos.

The Central Board of Secondary Education has been providing support to the academic needs of the learners worldwide. It has about 11500 schools affiliated to it and over 158 schools situated in more than 23 countries. The Board has always been conscious of the varying needs of the learners in countries abroad and has been working towards contextualizing certain elements of the learning process to the physical, geographical, social and cultural environment in which they are engaged. The International Curriculum being designed by CBSE-i, has been visualized and developed with these requirements in view.

The nucleus of the entire process of constructing the curricular structure is the learner. The objective of the curriculum is to nurture the independence of the learner, given the fact that every learner is unique. The learner has to understand, appreciate, protect and build on values, beliefs and traditional wisdom, make the necessary modifications, improvisations and additions wherever and whenever necessary.

The recent scientific and technological advances have thrown open the gateways of knowledge at an astonishing pace. The speed and methods of assimilating knowledge have put forth many challenges to the educators, forcing them to rethink their approaches for knowledge processing by their learners. In this context, it has become imperative for them to incorporate those skills which will enable the young learners to become 'life long learners'. The ability to stay current, to upgrade skills with emerging technologies, to understand the nuances involved in change management and the relevant life skills have to be a part of the learning domains of the global learners. The CBSE-i curriculum has taken cognizance of these requirements.

The CBSE-i aims to carry forward the basic strength of the Indian system of education while promoting critical and creative thinking skills, effective communication skills, interpersonal and collaborative skills along with information and media skills. There is an inbuilt flexibility in the curriculum, as it provides a foundation and an extension curriculum, in all subject areas to cater to the different pace of learners.

The CBSE has introduced the CBSE-i curriculum in schools affiliated to CBSE at the international level in 2010 and is now introducing it to other affiliated schools who meet the requirements for introducing this curriculum. The focus of CBSE-i is to ensure that the learner is stress-free and committed to active learning. The learner would be evaluated on a continuous and comprehensive basis consequent to the mutual interactions between the teacher and the learner. There are some non-evaluative components in the curriculum which would be commented upon by the teachers and the school. The objective of this part or the core of the curriculum is to scaffold the learning experiences and to relate tacit knowledge with formal knowledge. This would involve trans-disciplinary linkages that would form the core of the learning process. Perspectives, SEWA (Social Empowerment through Work and Action), Life Skills and Research would be the constituents of this ‘Core’. The Core skills are the most significant aspects of a learner’s holistic growth and learning curve.

The International Curriculum has been designed keeping in view the foundations of the National Curricular Framework (NCF 2005) NCERT and the experience gathered by the Board over the last seven decades in imparting effective learning to millions of learners, many of whom are now global citizens.

The Board does not interpret this development as an alternative to other curricula existing at the international level, but as an exercise in providing the much needed Indian leadership for global education at the school level. The International Curriculum would evolve on its own, building on learning experiences inside the classroom over a period of time. The Board while addressing the issues of empowerment with the help of the schools' administering this system strongly recommends that practicing teachers become skillful learners on their own and also transfer their learning experiences to their peers through the interactive platforms provided by the Board.

I profusely thank Shri G. Balasubramanian, former Director (Academics), CBSE, Ms. Abha Adams and her team and Dr. Sadhana Parashar, Head (Innovations and Research) CBSE along with other Education Officers involved in the development and implementation of this material.

The CBSE-i website has already started enabling all stakeholders to participate in this initiative through the discussion forums provided on the portal. Any further suggestions are welcome.

Vineet Joshi
Chairman
ACKNOWLEDGEMENTS

Advisory
Shri Vineet Joshi, Chairman, CBSE
Sh. N. Nagaraju, Director (Academic), CBSE

Conceptual Framework
Shri G. Balasubramanian, Former Director (Acad), CBSE
Ms. Abha Adams, Consultant, Step-by-Step School, Noida
Dr. Sadhana Parashar, Director (Training), CBSE

Ideators
Ms. Aditi Misra
Ms. Amita Mishra
Ms. Anita Sharma
Ms. Anita Makkar
Dr. Anju Srivastava

Ms. Anuradha Sen
Ms. Archana Sagar
Ms. Geeta Varshney
Ms. Guneet Ohri
Dr. Indu Khetrapal

Ms. Jaishree Srivastava
Dr. Kamla Menon
Dr. Meena Dhami
Ms. Neelima Sharma
Dr. N. K. Sehgal
Dr. Rajesh Hassija
Ms. Rupa Chakravarty
Ms. Sarita Manuja
Ms. Himani Asija
Dr. Uma Chaudhry

Material Production Groups: Classes IX-X

English :
Ms. Sarita Manuja
Ms. Renu Anand
Ms. Gayatri Khanna
Ms. P. Rajeshwary
Ms. Neha Sharma
Ms. Sarabjit Kaur
Ms. Ruchika Sachdev

Ms. Rachna Pandit
Ms. Neha Sharma
Ms. Sonia Jain
Ms. Dipinder Kaur
Ms. Sarita Ahuja

Dr. K.P. Chinda
Mr. J.C. Nijhawan
Ms. Rashmi Kathuria
Ms. Reemu Verma
Dr. Ram Avtar
Mr. Mahendra Shankar

Ms. Sharmila Bakshi
Ms. Archana Soni
Ms. Srilekha

Ms. Charu Maini
Ms. S. Anjum
Ms. Meenambika Menon
Ms. Novita Chopra
Ms. Neeta Rastogi
Ms. Pooja Sareen

Ms. Jayshree Srivastava
Ms. M. Bose
Ms. A. Venkatachalam
Ms. Smita Bhattacharya

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Ms. Priyanka Sen
Dr. Kavita Khanna
Ms. Keya Gupta

Ms. Meena Dhami
Mr. Saroj Kumar
Ms. Rashmi Ram singhaney
Ms. Seema Kapoor

Ms. Seema Rawat
Ms. N. Vidy
Ms. Mamta Goyal
Ms. Chhavi Raheja

Ms. Sheema Mathur
Ms. Seema Mathur
Ms. Kalpana Mathoo
Ms. Monika Thakur

Political Science:

Geography:

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Ms. Srilekha

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Ms. S. Anjum
Ms. Meenambika Menon
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Ms. Jayshree Srivastava
Ms. M. Bose
Ms. A. Venkatachalam
Ms. Smita Bhattacharya

Science :

Mathematics :

Geography:

History :

Ms. Charu Maini
Ms. S. Anjum
Ms. Meenambika Menon
Ms. Novita Chopra
Ms. Neeta Rastogi
Ms. Pooja Sareen

Ms. Jayshree Srivastava
Ms. M. Bose
Ms. A. Venkatachalam
Ms. Smita Bhattacharya

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Ms. Anuradha Mathur
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Mr. Bijoe Thomas

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Ms. Seema Chowdary
Ms. Ruba Chakravarty
Ms. Mahua Bhattacharya

Material Production Groups: Classes I-V

English :

Ms. S. Anjum
Ms. Meenambika Menon
Ms. Novita Chopra
Ms. Neeta Rastogi
Ms. Pooja Sareen

Material Production Group: Classes VI-VIII

English :

Science :

Mathematics :

Geography:

History :

Dr. Kavita Khanna
Ms. Priyanka Sen
Dr. Kavita Khanna
Ms. Keya Gupta

Dr. Meena Dhami
Mr. Saroj Kumar
Ms. Rashmi Ram singhaney
Ms. Seema Kapoor

Ms. Seema Rawat
Ms. N. Vidy
Ms. Mamta Goyal
Ms. Chhavi Raheja

Ms. Sharmila Bakshi
Ms. Archana Soni
Ms. Srilekha

Ms. Charu Maini
Ms. S. Anjum
Ms. Meenambika Menon
Ms. Novita Chopra
Ms. Neeta Rastogi
Ms. Pooja Sareen

Ms. Jayshree Srivastava
Ms. M. Bose
Ms. A. Venkatachalam
Ms. Smita Bhattacharya

Science :

Mathematics :

Geography:

History :

Ms. Ritu Narang, RO (Innovation)
Ms. Sindhu Saxena, R O (Tech)
Shri Al Hilal Ahmed, AEO

Ms. Seema Lakra, S O
Ms. Preeti Hans, Proof Reader

Coordinators:

Ms. Sugandh Sharma, E O (Com)
Ms. S. Anjum
Ms. Meenambika Menon
Ms. Novita Chopra
Ms. Neeta Rastogi
Ms. Pooja Sareen

Dr. Srijata Das, E O (Maths)
Dr. Rupak Chakravarty
Ms. Sarita Manuja
Ms. Himani Asija
Dr. Uma Chaudhry

Dr. Rashmi Sethi, E O (Science)
Shri R. P. Sharma, Consultant
Ms. Sharmila Bakshi
Ms. Archana Soni
Ms. Srilekha

Ms. Sheema Mathur
Ms. Seema Mathur
Ms. Kalpana Mathoo
Ms. Monika Thakur

Mr. Bijoe Thomas
Ms. Shilpi Anand
Ms. Nandita Mathur
Ms. Seema Chowdary
Ms. Ruba Chakravarty
Ms. Mahua Bhattacharya

Ms. Rupa Chakravarty
Ms. Sarita Manuja
Ms. Himani Asija
Dr. Uma Chaudhry

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Ms. Ritu Narang, RO (Innovation)
Ms. Sindhu Saxena, R O (Tech)
Shri Al Hilal Ahmed, AEO

Ms. Seema Lakra, S O
Ms. Preeti Hans, Proof Reader

Coordinators:

Dr. Sadhana Parashar, Head (I and R)
Ms. Sugandh Sharma, E O (Com)
Ms. S. Anjum
Ms. Meenambika Menon
Ms. Novita Chopra
Ms. Neeta Rastogi
Ms. Pooja Sareen

Dr. Srijata Das, E O (Maths)
Dr. Rashmi Sethi, E O (Science)
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Ms. Mahua Bhattacharya

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Ms. Himani Asija
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Ms. Monika Thakur
Mr. Bijoe Thomas

Ms. Nandita Mathur
Ms. Seema Chowdary
Ms. Ruba Chakravarty
Ms. Mahua Bhattacharya

Ms. Ritu Narang, RO (Innovation)
Ms. Sindhu Saxena, R O (Tech)
Shri Al Hilal Ahmed, AEO

Ms. Seema Lakra, S O
Ms. Preeti Hans, Proof Reader
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## Syllabus

**Arithmetic Progression and Geometric Progression (core)**

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<th>Topic</th>
<th>Description</th>
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<tr>
<td>Introduction to Arithmetic Progression (A.P.) and Geometric Progression (G.P.)</td>
<td>Recalling number patterns and geometrical patterns, Illustration of arithmetic progression from daily life situations, terms of A.P., first term, common difference, terms of G.P., first term, common ratio</td>
</tr>
<tr>
<td>General term of an A.P.</td>
<td>nth term of A.P. as $a_n = a + (n-1)d$, where ‘$a$’ is first term and ‘$d$’ is common difference, $n$ is total number of terms</td>
</tr>
<tr>
<td>General term of a G.P.</td>
<td>nth term of G.P. as $a_n = ar^{n-1}$, where ‘$a$’ is first term, ‘$r$’ is common ratio and $a_n$ is the required term</td>
</tr>
<tr>
<td>Sum of first $n$ terms of A.P.</td>
<td>$S_n = (n/2)[2a + (n-1)d]$, finding unknown when any three of $a$, $n$, $d$, $S_n$ are given</td>
</tr>
</tbody>
</table>
**Scope document (Core)**

**Learning Objectives:**

- Observe number patterns and guess the next term
- Observe geometrical patterns and draw the next in turn
- Recognise Arithmetic progression and read its first term and common difference
- Write the nth term or general term of a given A.P. in the form $a_n = a + (n-1) \ d$, where ‘$a$’ is the first term and ‘$d$’ is the common difference.
- Understand that the general term of an A.P. is always a linear expression.
- Write the specified term of an A.P. when $a$, $n$ and $d$ are known
- Determine the unknown quantity in the expression $a_n = a + (n-1) \ d$ under given conditions
- Recognise Geometric progression and read its first term and common ratio
- Write the nth term or general term of a given G.P. in the form $a_n = ar^{n-1}$, where $a$ is the first term and $r$ is the common ratio.
- Write the specified term of a G.P. when $a$, $n$ and $r$ are known
- Determine the unknown quantity in the expression $a_n = ar^{n-1}$ under given conditions
- Understand that sum of first $n$ terms of an A.P. is $S_n = \frac{n}{2} [2a + (n-1) \ d]$
- Determine the sum of first $n$ terms of an A.P. for specified value of $n$
  a) When $a$, $n$ and $d$ are known
  b) When $S_n$ is known for specified value of $n$.
- Determine the unknown quantity in the expression $S_n = \frac{n}{2} [2a + (n-1) \ d]$ under given conditions

**Cross-curricular links:**

1. Sequences generate the musical notes and hence fill the life with rhythm. ‘Seven Sawar’ in Indian classical music produce around divine vibrations when all ‘sawar’ are arranged in a sequence in the form of different Ragas.
Mozart the finest musician of the world has adopted Fibonacci sequences in best of his compositions. Read more about Fibonacci Music on the following link:

http://techcenter.davidson.k12.nc.us/group2/music.htm

2. Observe Physical Fitness classes in your school. Enjoy the sequences of drills, patterns formed by movements of body.......and their impact on health.

3. For knowing more about use of sequences in art, architecture, poetry...... visit the following link:

http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibInArt.html#daVinci

**Extension Activities:**

Visit the given link and explore methods of geometrical designing using number sequences.

http://www.mathsmaster.org/home/sequence-designs/

**SEWA:**

- If you start saving Rs 500 in a month and decide to save Rs 100 more in each subsequent month, how much money you will be able to save in 10 years?

**Research:**

Techniques of finding sum of first 100 natural numbers

http://betterexplained.com/articles/techniques-for-adding-the-numbers-1-to-100/
Teacher’s Support Material
Teacher’s Note

The teaching of Mathematics should enhance the child’s resources to think and reason, to visualize and handle abstractions, to formulate and solve problems. As per NCF 2005, the vision for school Mathematics includes:

1. Children learn to enjoy mathematics rather than fear it.
2. Children see mathematics as something to talk about, to communicate through, to discuss among them, to work together on.
3. Children pose and solve meaningful problems.
4. Children use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth or falsity of statements.
5. Children understand the basic structure of Mathematics: Arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathematics, all offer a methodology for abstraction, structuration and generalisation.
6. Teachers engage every child in class with the conviction that everyone can learn mathematics.

Students should be encouraged to solve problems through different methods like abstraction, quantification, analogy, case analysis, reduction to simpler situations, even guess-and-verify exercises during different stages of school. This will enrich the students and help them to understand that a problem can be approached by a variety of methods for solving it. School mathematics should also play an important role in developing the useful skill of estimation of quantities and approximating solutions. Development of visualisation and representations skills should be integral to Mathematics teaching. There is also a need to make connections between Mathematics and other subjects of study. When children learn to draw a graph, they should be encouraged to perceive the importance of graph in the teaching of Science, Social Science and other areas of study. Mathematics should help in developing the reasoning skills of students. Proof is a process which encourages systematic way of argumentation. The aim should be to develop arguments, to evaluate arguments, to make conjunctures and understand that there are various methods of reasoning. Students should be made to understand that mathematical communication is
precise, employs unambiguous use of language and rigour in formulation. Children should be encouraged to appreciate its significance.

At the secondary stage students begin to perceive the structure of Mathematics as a discipline. By this stage they should become familiar with the characteristics of Mathematical communications, various terms and concepts, the use of symbols, precision of language and systematic arguments in proving the proposition. At this stage a student should be able to integrate the many concepts and skills that he/she has learnt in solving problems.

The unit on Arithmetic Progression focuses on observation and generalisation in order to achieve the following learning objectives:

Arithmetic or geometric or any other sequence is basically the mathematical expression of patterns occurred in nature, in geometrical designs, in textile designs, in music compositions etc. Thus the unit has been introduced through observations of patterns in number sequences and in geometrical patterns. Warm up activities lights up the mind bulb to enable the prediction of the next terms of the sequences or the next shape in the design. Warm up activity W3 helps the students to establish the relation between number sequences and geometrical patterns. The purpose is to make them translate the geometrical sequences into number sequences.

Pre-content activities set the pace for use of symbols and rules to generalize the inferences drawn from observations. So the learners will use algebraic expressions to write the general term of any given sequence. Practice of lots of number sequences can be given for the same. For example,

2, 4, 6...
3, 7, 10........
1, 4, 9, 16....... 
4, 7, 12, 19...... 
½, 1/2², ½³........

Students can write the general terms for these sequences. Also they can be given the general terms and asked to write the first few terms of the sequence. They shall also
realise that if general term is given they can write the specified terms without writing all terms in continuation. For example if general term $t_n = 5n+6$, it is possible to write 100th term by substituting $n= 100$ in $t_n$.

Further the students can be asked to observe the similarities between the following sequences:
2, 4, 6, 8.....
4, 7, 10....
11, 16, 21....

Let them come out with the observation that in each case the difference between the consecutive terms is same. For such sequence there is special name i.e. Arithmetic sequence. Emphasis is to be laid on building the vocabulary of this unit as all the terms are new for the students – first term, common difference, arithmetic sequence, arithmetic progression etc.. In the same way geometric progression and related terms can be introduced.

Teacher should take care that learner is able to use the vocabulary and symbols carefully as this chapter also laid the foundation for functions. They shall further be encouraged to explore that terms of all arithmetic progressions observe linear relation and geometric progression observe exponential relation. By observing the pattern in numbers students shall be able to get the general term as $a_n = a+(n-1)d$ for A.P. and $a_n = ar^{n-1}$ for G.P.

Sum of first $n$ terms can be introduced using GAUSS METHOD. Following incident from life of Gauss can be shared to motivate them-

One day teacher of Gauss gave a problem to the students in order to engage them for a long time. Problem was to find the sum of first 100 numbers. Gauss got the solution in 20 seconds and surprised his teacher. What was the method?

He Wrote 
\[
(1+2+3+\ldots+100) \\
(100+99+98+\ldots+1) \\
\frac{101\times100}{101+101+\ldots+101} = 10100
\]
Students shall be motivated to find the other innovative method of finding the sum of first 100 natural numbers.

Moreover, they shall be encouraged to try this with more numbers and finally to generalize for any arithmetic sequence. Students shall be able to find the formula for sum of first term of A.P.

At the same time teacher can show that the formula for sum of first n terms of A.P. contains four variable n, a, d, and \( S_n \) in

\[
S_n = \frac{n}{2} [2a + (n - 1)d]
\]

If any of the variable is unknown, with the help of given information, it can be determined.

Gradually, teacher can take the students to the type of problems where two variables are known and the information in the problem can be used to track the two unknown variables.

Same shall be done with the problems of G.P.

Students shall also be encouraged to frame such problems on their own. This practice will help them to internalize all concepts learnt, strongly.
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<thead>
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<th>Type of Activity</th>
<th>Name of Activity</th>
<th>Skill to be developed</th>
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<td>Recognizing number patterns.</td>
<td>Observation, analytic</td>
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<td>Warm UP(W2)</td>
<td>Recognizing patterns in shapes</td>
<td>Observation, analytic</td>
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<tr>
<td>Warm UP(W3)</td>
<td>Designs and numbers</td>
<td>Observation, analytic, making connections</td>
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<td>Pre-Content (P1)</td>
<td>Match stick activity</td>
<td>Observation, thinking skills, analytic skills</td>
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<td>Pre-Content (P2)</td>
<td>Visualizing specified term</td>
<td>Observation Analytical skills and synthesis</td>
</tr>
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<td>Content (CW1)</td>
<td>nth term of an A.P.</td>
<td>Observation ,Analytical skills ,inferential, synthesis</td>
</tr>
<tr>
<td>Content (CW2)</td>
<td>Recognizing an A.P.</td>
<td>Analytical, inferential, synthesis</td>
</tr>
<tr>
<td>Content (CW3)</td>
<td>Terms of an A.P.</td>
<td>Analytical, inferential, synthesis</td>
</tr>
<tr>
<td>Content (CW4)</td>
<td>Problems on A.P.</td>
<td>Thinking skill, problem solving</td>
</tr>
<tr>
<td>Content (CW5)</td>
<td>Hands on activity 1</td>
<td>Observation, Inferential skill, Thinking Skill, Application.</td>
</tr>
<tr>
<td>Content (CW6)</td>
<td>Recognizing a G.P.</td>
<td>Analytical, inferential, synthesis</td>
</tr>
<tr>
<td>Content (CW7)</td>
<td>General term of a G.P.</td>
<td>Analytical, inferential, synthesis</td>
</tr>
<tr>
<td>Content (CW8)</td>
<td>Problems on G.P.</td>
<td>Thinking, problem solving</td>
</tr>
<tr>
<td>Content (CW9)</td>
<td>Sum of first n terms of an A.P.</td>
<td>Observation, Analytical, inferential, synthesis</td>
</tr>
<tr>
<td>Content (CW10)</td>
<td>Hands on Activity</td>
<td>Observation, Inferential skill, Thinking</td>
</tr>
<tr>
<td>Content (CW 11)</td>
<td>Problems on sum of first n terms of A.P.</td>
<td>Thinking skill, problem solving</td>
</tr>
<tr>
<td>-------------------------</td>
<td>------------------------------------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>Post - Content (PCW 1)</td>
<td>Assignment</td>
<td>Problem solving skills, Application.</td>
</tr>
<tr>
<td>Post - Content (PCW 2)</td>
<td>Brainstorming</td>
<td>Conceptual understanding, Reasoning skills.</td>
</tr>
<tr>
<td>Post - Content (PCW 3)</td>
<td>Creating designs using A.P.</td>
<td>Application, Creativity</td>
</tr>
<tr>
<td>Post - Content (PCW 4)</td>
<td>Framing questions</td>
<td>Thinking skills, Creativity</td>
</tr>
<tr>
<td>Post-Content (PCW5)</td>
<td>Hands on activity</td>
<td>Visualisation</td>
</tr>
</tbody>
</table>
Activity 1 – Warm Up (W1)
Recognising Number Patterns

Specific objective: To observe the given number pattern and generate next term.

Description: Students are aware of number sequences. This is a warm up activity to gear up the students to observe sequences and patterns in them (if any).

Execution: In the beginning of the lesson, distribute SW1 and ask the students to observe the number pattern. You may prepare flashcards and show them to students.

Parameters for assessment:

- Able to observe number pattern (if any)
- Able to generate the next terms using pattern

Activity 2 – Warm Up (W2)

Recognising Patterns in Shapes

Specific objective: To observe the pattern in geometrical design and to create new pattern.

Description: This activity is designed for understanding the use of patterns in creating designs.

Execution: Students will be provided with a squared sheet on which they will be asked to draw a design using pattern.

Parameters for assessment:

- Able to observe patterns in shapes
- Able to generate the next in turn using pattern
Activity 3 – Warm Up (W3)
Designs and Numbers

Specific objective: To draw the next shape by observing the pattern.

Description: In this activity students will observe the sequence of shapes and will draw the next in turn.

Execution: Distribute W3 and let students use different colours for making next in turn.

Parameters for assessment:

- Able to observe pattern in shapes (if any)
- Able to generate next in turn using pattern
- Able to associate patterns and numbers

Extra reading: http://www.mathsisfun.com/numberpatterns.html
Activity 4 – Pre Content (P1)

Match Sticks Activity

Specific objective: To associate patterns with numbers.

Description: This is a match stick activity for relating patterns with numbers.

Execution: Students will be asked to make patterns using match sticks and count the number of match sticks used. They will be further encouraged to observe the number of matchsticks required with the addition of one repeated shape. Further, they will be motivated to find the number of matchsticks required for rows containing more number of repeated shapes.

Parameters for assessment:

- Able to make a sequence of numbers using patterns
- Able to generate the specified term using the pattern

Extra reading:
Activity 5 – Pre Content (P2)
Visualising Specified Term

Specific objective: To visualize the specified term in the given pattern.

Description: This activity is designed to motivate the students to visualize the specified term in a given pattern sequence.

Execution: Show the students the following diagram and initiate a discussion on number patterns and specified terms. Ask them to do the worksheet.

Parameters for assessment:

- Able to find the specified term.
Activity 6 – Content (CW1)

nth Term of an A.P

\[ T_n = a + (n - 1) \cdot d \]

Specific objective: To write the nth term of an A.P.

Description: Through this activity sheet students would be able to find the formula for general term of an A.P. as \( a_n = a + (n-1) \cdot d \)

Execution: Ask the students to observe the pattern given in the worksheet. Motivate them to observe the pattern and generate nth term. Using the formula for nth term, ask them to find specified terms like 20th term, 100th term etc.

Note: Discuss the concept of finding nth term from the end.

Parameters for assessment:

- Able to write the formula for nth term of an A.P.
- Able to write a general A.P.

Extra reading:

Activity 7 – Content (CW2)

Recognising an A.P

Specific objective: To recognize an A.P from given sequences.

Description: This activity has been designed to make students learn the methods of recognizing an A.P out of given sequences. Students will explore two methods. The first one by finding common difference using \(a_2 - a_1\) and \(a_3 - a_2\). Also, they would be encouraged to observe that \(a_n - a_{n-1}\) is a constant in every A.P.

Execution: Let students explore and investigate on their own. Distribute the worksheets and give some time for investigation. After some time, discuss the observations in groups.

Parameters for assessment:

<table>
<thead>
<tr>
<th>able to tell whether a given sequence is an A.P. or not</th>
</tr>
</thead>
<tbody>
<tr>
<td>able to write the nth term of given sequence.</td>
</tr>
<tr>
<td>able to predict about the common difference, given the nth term.</td>
</tr>
</tbody>
</table>

Extra reading:

Activity 8 – Content (CW3)

Terms of an A.P

Specific objective: To explore that the nth term of an A.P. is a linear expression and common difference of an A.P is free from n.

Description: Students have fair idea of A.P, vocabulary related to A.P and its general term. Through this activity students will explore various other facts regarding the general term.

Execution: Students will be given a worksheet containing problems based on general term of A.P. While attempting these problems students will explore the fact that the nth term of an A.P. is a linear expression. They will investigate that common difference of an A.P is always a constant.

Parameters for assessment:

- Able to write the specified term when nth term of an A.P. is given
- Able to understand that linear expressions forms an A.P.
- Able to find that common difference of an A.P is always a constant
Activity 9 – Content (CW4)
Problems on A.P.

Specific objective: To apply the knowledge of A.P. in problem solving.

Description: This is a problem worksheet which focuses on the application of knowledge of general term in finding the unknown variable.

Execution: The worksheet may be given to students as an assignment.

Students can be given prescribed tome to complete the assignment. Problems of the assignment can be discussed in the class after checking the assignment. Various problem solving strategies used by the students can be discussed in the class.

Parameters for assessment:

- Able to find the specified term of an A.P.
- Able to find the number of terms in a given A.P.
- Able to find whether a given term belongs to the given A.P.
- Able to find the given term is which term of the given A.P.
- Able to find the nth term from the end of the given A.P.

Extra reading: http://cristina327.hubpages.com/hub/Solving-Arithmetic-Sequences
Activity 10 – Content (CW5)

Hands on Activity 1

Specific objective: To check whether the given sequence is an AP or not by paper cutting and pasting.

Description: This activity has been designed to visualise the sequences representing an A.P.

Execution : Ask the students to bring the materials required for the activity. Distribute the worksheet and discuss the observations in the end.

Parameters for assessment:

- Able to visualise an A.P using a paper model.
Activity 11 – Content (CW6)

Recognising a G.P.

Specific objective: To recognise a G.P.

Description: This activity worksheet is designed to explore the geometric progressions.

Execution: Students will be asked to investigate the given sequences and separate G.P. by finding the ratios of two consecutive terms.

Parameters for assessment:

- Able to find the ratio of consecutive terms
- Able to recognise a G.P.
- Able to tell whether a given sequence is a G.P. or not
- Able to write a G.P. with given first term and common ratio
**Activity 12 – Content (CW7)**

**General Terms of a G.P.**

**Specific objective:** To write the nth term of a G.P.

**Description:** Through this activity sheet students would be able to find the formula for general term of a G.P. as \( a_n = ar^{n-1} \)

**Execution:** Ask the students to observe the pattern given in the worksheet. Motivate them to observe the pattern and generate nth term. Using the formula for nth term, ask them to find specified terms like 20th term, 100th term etc.

**Parameters for assessment:**

- Able to write the formula for nth term of a G.P.
Activity 13 – Content (CW8)

Problems on G.P.

Specific objective: To apply the knowledge of G.P. in problem solving.

Description: This is a problem worksheet which focuses on the application of knowledge of general term in finding the unknown variable.

Execution: The worksheet may be given to students as an assignment.

Students can be given prescribed tome to complete the assignment. Problems of the assignment can be discussed in the class after checking the assignment. Various problem solving strategies used by the students can be discussed in the class.

Parameters for assessment:

- Able to find the specified term of an G.P.
Activity 14 – Content (CW9)

Sum of n Terms of an A.P

\[ s_n = \frac{n}{2} [2a + (n - 1)d] \]

\[ = \frac{n}{2} [a + (a + (n - 1)d)] \]

\[ = \frac{n}{2} (a + t_n) \]

**Specific objective:** To find the sum of first n terms of an A.P

**Description:** Through this activity sheet, students will find the formula for the sum of first n terms of an A.P. A few steps are given as a hint.

**Execution:** Initiate the discussion with students on how Gauss found the sum of first 100 natural numbers. Proceeding further, students will be asked to find the formula for the sum of first n terms of an A.P using the clues given in the worksheet. Further, they would be asked to find the solutions of problems using the formulae.

**Parameters for assessment:**

- Able to find the formulae for finding the sum of first n terms of an A.P.

Activity 15 – Content (CW10)

Hands on Activity 2

**Specific objective:** To verify that the sum of first \( n \) odd natural numbers is \( n^2 \) using visualisation.

**Description:** This is a hands on activity to visualise the sum of first \( n \) odd natural numbers.

**Execution:** Students will be given the instruction sheet. They will perform the activity individually and share their observations.

**Parameters for assessment:**

- Able to perform the hands on activity as per the instructions.
- Able to verify the result.
Activity 16 – Content (CW11)

Problems on Sum of First n Terms of an A.P

Specific objective: To apply the formula for finding the sum of first n terms of an A.P

Description: Through this activity sheet students will be exposed to problems involving the use of formula in finding the sum of first n terms of an A.P

Execution: Discuss the various ways of finding the sum of n terms of an A.P. Ask the students to solve the problems given in the worksheet. Discuss the ways of solving the problems.

Parameters for assessment:

- Able to find the sum of first n terms of an A.P. when first term and common difference is given.
- Able to find the sum of first n terms of an A.P. when first term, second term and last term is given.
- Able to find the sum of first n terms of an A.P. when any two of its terms are given.
- Able to find the sum of first n terms of an A.P. under given condition.

Activity 17 – Post Content (PCW1)

Assignment

Activity 18 – Post Content (PCW2)

Brainstorming

Activity 19 – Post Content (PCW3)

Creating designs using A.P.

Activity 20 – Post Content (PCW4)

Framing questions

Activity 21 – Post Content (PCW5)

Hands on activity
Assessment Guidance Plan for Teachers

With each task in student support material a self-assessment rubric is attached for students. Discuss with the students how each rubric can help them to keep in tune their own progress. These rubrics are meant to develop the learner as the self motivated learner.

To assess the students’ progress by teachers two types of rubrics are suggested below, one is for formative assessment and one is for summative assessment.

Suggestive Rubric for Formative Assessment (exemplary)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mastered</th>
<th>Developing</th>
<th>Needs motivation</th>
<th>Needs personal attention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the general term of an A.P.</td>
<td>• Able to recognize that given sequence is an A.P. and can write its first term and common difference</td>
<td>• Able to recognize that given sequence is an A.P. and can write its first term and common difference</td>
<td>• Able to recognize that given sequence is an A.P. and can write its first term and common difference</td>
<td>• Able to recognize that given sequence is an A.P. and can write its first term and common difference</td>
</tr>
<tr>
<td></td>
<td>• Able to write general term of A.P. as ( t_n = a + (n-1)d )</td>
<td>• Able to write general term of A.P. as ( t_n = a + (n-1)d )</td>
<td>• Able to write general term of A.P. as ( t_n = a + (n-1)d )</td>
<td>• Not able to write general term of A.P. as ( t_n = a + (n-1)d )</td>
</tr>
<tr>
<td>• Able to write general term when any two terms (specified) are given by determining its first term and common difference</td>
<td>• Able to write general term when any two terms (specified) are given by determining its first term and common difference</td>
<td>• Not able to write general term when any two terms (specified) are given. Can determine its first term and common difference partially correct.</td>
<td>• Not able to write general term when any two terms (specified) are given by determining its first term and common difference</td>
<td></td>
</tr>
<tr>
<td>Able to write general term when sum of first n terms are given, using the formula ( S_n - S_{n-1} = t_n )</td>
<td>Not able to write general term when sum of first n terms are given, using the formula ( S_n - S_{n-1} = t_n ).</td>
<td>Not able to write general term when sum of first n terms are given, using the formula ( S_n - S_{n-1} = t_n ).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the above rubric it is very clear that
- Learner requiring personal attention is poor in concepts and requires the training of basic concepts before moving further.
- Learner requiring motivation has basic concepts but face problem in calculations or in making decision about suitable substitution etc. He can be provided with remedial worksheets containing solution methods of given problems in the form of fill-ups.

- Learner who is developing is able to choose suitable method of solving the problem and is able to get the required answer too. May have the habit of doing things to the stage where the desired targets can be achieved, but avoid going into finer details or to work further to improve the results. Lerner at this stage may not have any mathematical problem but may have attitudinal problem. Mathematics teachers can avail the occasion to bring positive attitudinal changes in students’ personality.

- Learner who has mastered has acquired all types of skills required to solve the problems based on general formula of A.P. and G.P.
### Teachers’ Rubric for Summative Assessment of the Unit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognizing sequence</td>
<td>• Able to recognize the pattern in a given number sequence</td>
<td></td>
<td></td>
<td></td>
<td>• Able to recognize the pattern in a given number sequence</td>
</tr>
<tr>
<td></td>
<td>• Able to identify A.P. and G.P. and can write its first term and common difference or common ratio</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to identify A.P. and G.P. and not able to write its first term and common difference or common ratio</td>
</tr>
<tr>
<td></td>
<td>• Able to write the next few terms</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to write the next few terms</td>
</tr>
<tr>
<td></td>
<td>• Able to recognize the sequence when general term of a sequence is given</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to recognize the sequence when general term of a sequence is given</td>
</tr>
<tr>
<td>General term of an A.P.</td>
<td>• Able to write the general term of an A.P. when ‘a’ and ‘d’ are given’</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to write the general term of an A.P. when ‘a’ and ‘d’ are given’</td>
</tr>
<tr>
<td></td>
<td>• Able to write general term by finding ‘a’ and ‘d’ when any two</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to write general term by</td>
</tr>
<tr>
<td>Sum of n terms of an A.P terms are given</td>
<td>Able to find the value of an unknown variable when three of a,d,n, t_n are given using the formula of general term.</td>
<td></td>
<td></td>
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<tr>
<td>-----------------------------------------</td>
<td>----------------------------------------------------------------------------------------------------</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Able to write specified term when general term is given.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Able to determine specified term using the given information/conditions.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Able to prove that sum of first n terms of an A.P. is ( S_n = \frac{n}{2}[2a+(n-1)d] )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Able to find the sum of first n terms when a and d are given using formula.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>finding ‘a’ and ‘d’ when any two terms are given</td>
<td>Not able to find the value of unknown variable when three of a,d,n, t_n are given using the formula of general term.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not able to write specified term when general term is given.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not able to determine specified term using the given information/conditions.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not able to prove that sum of first n terms of an A.P. is ( S_n = \frac{n}{2}(2a+(n-1)d) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Not able to find the sum of first n terms when a</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
• Able to find the sum of given number of terms starting from first term.

• Able to find the value of unknown variable when three of a, r, n, S_n are given using the formula of sum of first n terms.

• Able to write the general term \( t_n = S_{m+n} - S_m \) when sum of first \( m+n \) terms and sum of first \( n \) terms are given.

• Able to solve multistep problems based on formulae of general terms and sum of first \( n \) terms accurately.

• Not able to find the sum of given number of terms starting from first term accurately.

• Not able to find the value of unknown variable when three of a, r, n, S_n are given using the formula of sum of first \( n \) terms accurately.

• Not able to write the general term \( t_n = S_{m+n} - S_m \) when sum of first \( m+n \) terms and sum of first \( n \) terms are given.

• Not able to solve multistep problems based on formulae of general terms and sum of first
<table>
<thead>
<tr>
<th>General term of G.P.</th>
<th>• Able to write the general term of a G.P. when ‘a’ and ‘r’ are given.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Able to write general term by finding ‘a’ and ‘r’ when any two terms are given</td>
</tr>
<tr>
<td></td>
<td>• Able to find the value of unknown variable when three of $a, r, n, t_n$ are given using the formula of general term.</td>
</tr>
<tr>
<td></td>
<td>• Able to write specified term when general term is given.</td>
</tr>
<tr>
<td></td>
<td>• Able to determine specified term using the given information/conditions.</td>
</tr>
</tbody>
</table>

| n terms accurately. |
|---------------------|---------------------------------------------------------------------|
| • Not able to write the general term of a G.P. when ‘a’ and ‘r’ are given. |
| • Not able to write general term by finding ‘a’ and ‘r’ when any two terms are given |
| • Not able to find the value of unknown variable when three of $a, r, n, t_n$ are given using the formula of general term. |
| • Not able to write specified term when general term is given. |
| • Not able to determine specified term using the given information/conditions. |
STUDY MATERIAL
Arithmetic Progression and Geometric Progression

Introduction:

Earlier, you have come across a number of patterns such as

(i) \[1^2 = 1\]
\[11^2 = 121\]
\[111^2 = 12321\]
\[1111^2 = 1234321\]
\[\vdots\]

(ii) \[7^2 = 49\]
\[67^2 = 4489\]
\[667^2 = 444889\]
\[6667^2 = 44448889\]
\[\vdots\]

(iii) \[11^2 = 121\]
\[101^2 = 10201\]
\[1001^2 = 1002001\]
\[10001^2 = 100020001\]
\[100001^2 = 10000200001\]

(iv) Number of dots: 1, 4 = 2^2, 9 = 3^2, 16 = 4^2, 25 = 5^2, \ldots
M is the midpoint of the side of first square, $M'$ is the midpoint of the side of the second square and so on.

Area of 1st square = $a^2$

Area of 2nd square = $\frac{a^2}{2}$

Area of 3rd square = $\frac{a^2}{4} = \frac{a^2}{2^2}, \ldots$

\[
\begin{align*}
\text{(vii)} & \quad \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots \\
\text{(viii)} & \quad 1 \times 3, 3 \times 6, 5 \times 12, 7 \times 24, \ldots \\
\text{(ix)} & \quad 1, 1, 2, 3, 5, 8, 13, 21, \ldots \\
\text{(x)} & \quad \sqrt{2}, 3, \sqrt{4}, 5\sqrt{6}, 7, \ldots
\end{align*}
\]
In some of these patterns, we can guess and decide the next number of the pattern while in some others we are not able to.

In this unit, we shall recall some of these patterns and try to arrive at the next number in each pattern. In particular, we will study some pattern of the numbers in which

(i) two consecutive terms differ by a constant,
(ii) the ratio of two consecutive terms is the same.

We shall also study the applications of these two types of patterns in daily life.

1. **Recognition of Patterns:**

   Look at the patterns given above and try to write next number in each pattern.

   (i) How will you find 111111\(i\) e., the sixth row?
       Observing the patterns, we can write it as 111111\(i\) = 12345654321
       Can you write 111111\(i\)?
       Can you write 1111111\(i\)?
       Can you also write 111111111\(i\)? Try!!

   (ii) Observing the pattern, we can write 666667\(i\) as
       666667\(i\) = 444444 888889
       Similarly,
       666667\(i\) = 444444 888889

   (iii) Observing the patterns, we can write 1000001\(i\) as
       1000001\(i\) = 1000002 000001
       Similarly,
       1000001\(i\) = 1000002 000001

   (iv) Number of dots in the next design = 6\(i\) and the number of dots can also be expressed as 1+3+5+7+9+11
       Similarly, in the next design, the number of dots = 7\(i\) and this number can be expressed as 1+3+5+7+9+11+13

   (v) Number of lines in the next circular design = 16 and the number of parts = 2\(i\)
(vi) Area of fourth square = $\frac{a^2}{2^3}$

Similarly, area of fifth square = $\frac{a^2}{2^4}$ and area of 6th square = $\frac{a^2}{2^5}$ and so on.

Area of the nth square = $\frac{a^2}{2^{n-1}}$, where n is a natural number.

(vii) Next number in the pattern = $\frac{6}{7}$

Similarly, next to this number = $\frac{7}{8}$ and so on.

Clearly, the nth number = $\frac{n}{n+1}$, where n is a natural number.

(viii) Observing the pattern, the next number = 9 x 48 [Next number of 1, 3, 5, 7 is 9 (Why?) and that of 3, 6, 12, 24 is 48 (Why?)]

(ix) Here, we see that

1st number = 1
2nd number = 1
3rd number = 1+1=2
4th number = 1+2=3
5th number = 2+3=5
6th number = 3+5=8
7th number = 5+8=13
8th number = 8+13=21
9th number = 13+21=34
10th number = 21+34 = 55 and so on.

Here, every number, except the first two numbers, is the sum of preceding two numbers.

This pattern is known as Fibonacci Sequence, named after a great Italian mathematician Leonardo Fibonacci (1170-1250).

(x) Here, 1st number = $\sqrt{2}$
2nd number = 3 = 2+1
3rd number = \(\sqrt{4} = \sqrt{2 \times 2}\)
4th number = 5 = 4+1
5th number = \(\sqrt{6} = \sqrt{2 \times 3}\)
6th number = 7 = 6+1
Now, 7th number = \(\sqrt{8} = \sqrt{2 \times 4}\)
8th number = 8+1 = 9
and so on.

Let us consider some more examples:

**Example 1:** Observe the following geometric pattern and write the number of dots in the next cube.

Solution: We get the following pattern of dots:

\[1^3, 2^3, 3^3, \ldots\]

So, number of dots in the next cube will be \(4^3 = 64\)

**Example 2:** Find the next number in the following patterns:

(i) 1, 2, 3, 4, 5, __________
(ii) 1, 3, 5, 7, 9, __________
(iii) 1, 2, 4, 8, ______________

(iv) \[
\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \ldots
\]

(v) 1, 4, 9, 16, 25, 36, __________

(vi) 1, 2, 6, 24, 120, ____________

(vii) \[
\frac{1}{3}, \frac{1}{8}, \frac{1}{13}, \frac{1}{18}, \ldots
\]

**Solution:**

(i) Clearly, the next number will be 6

(ii) Next number = 9 + 2 = 11. Each number except first number is obtained by adding 2 to its preceding number.

(iii) Next number = 16. Each number except first number is obtained by multiplying its preceding number by 2.

(iv) Next number = \(\frac{1}{48}\). Each number, except the first number, is obtained by multiplying its preceding one by \(\frac{1}{2}\).

(v) \[1 = 1^2, \quad 4 = 2^2, \quad 9 = 3^2, \quad 16 = 4^2, \quad 25 = 5^2 \quad 36 = 6^2.\]

So, the next number = \(7^2 = 49\)

(vi) Here, 1 = 1

\[
2 = 2 \times 1 \\
6 = 3 \times 2 \times 1 \\
24 = 4 \times 3 \times 2 \times 1 \\
120 = 5 \times 4 \times 3 \times 1
\]

So, the next number will be \(6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720\)

**The product** 2 x 1 **is also written as 2! and read as 2 factorial. 3 x 2 x 1 is written as 3! and read as 3 factorial.**

Similarly, 6 x 5 x 4 x 3 x 2 x 1 is written as 6!
(vii) We have
\[
\frac{1}{3} = \frac{1}{3}
\]
\[
\frac{1}{8} = \frac{1}{3+5}
\]
\[
\frac{1}{13} = \frac{1}{8+5}
\]
\[
\frac{1}{18} = \frac{1}{13+5}
\]
So, the next term = \(\frac{1}{18+5} = \frac{1}{23}\)

**Example 3:** Write the next two numbers in the following patterns:

(i) \(\frac{2}{5}', \frac{9}{15}', \frac{16}{45}' \ldots, \ldots\)

(ii) 54, 18, 6, 2, \(\frac{2}{3}', \ldots, \ldots\)

(iii) 100, 95, 90, 85, \ldots, \ldots

**Solution:**

(i) we have
\[
\frac{2}{5} = \frac{2}{5},
\]
\[
\frac{9}{15} = \frac{2+7}{5\times3}, \frac{16}{45} = \frac{9+7}{15\times3}
\]
So, next two numbers are \(\frac{16+7}{45\times3}, \text{i.e.,} \frac{23}{135}\)

and \(\frac{23+7}{135\times3} = \frac{30}{405}\)

Hence, the next two numbers are \(\frac{23}{135}\) and \(\frac{30}{405}\) respectively.
(ii) \[ \begin{align*} 54 &= 54 \\ 18 &= \frac{54}{3} = 54 + 3 \\ 6 &= \frac{18}{3} = 18 ÷ 3 \\ 2 &= \frac{6}{3} = 6 ÷ 3 \\ \frac{2}{3} &= \frac{2}{3} = 2 ÷ 3 \end{align*} \]

Next number \[= \frac{2}{3} + 3 = \frac{2}{9} \]

and next number to \[= \frac{2}{9} + 3 = \frac{2}{27} \]

(iii) \[\begin{align*} 100 &= 100 \\ 95 &= 100 - 5 \\ 90 &= 95 - 5 \\ 85 &= 90 - 5 \end{align*} \]

So, the next two numbers will be 85 - 5, i.e., 80 and 80 - 5, i.e., 75 respectively.

2. **Arithmetic Progression**

Let us revisit Example 2 [pattern (i) and (ii)] and Example 3 [pattern (ii)]. We reproduce them as follows:

(i) \[1, 2, 3, 4, 5, \ldots \]

(ii) \[1, 3, 5, 7, 9, \ldots \]

(iii) \[100, 95, 90, 85, \ldots \]

In pattern (i), each number 1,2,3,4,5 is called a **term**

Here, \[1\] is first term, \[2\] is the second term, \[3\] is the third term and so on.
Similarly in (ii), 1 is the first term, 3 is the second term, 5 is the third term, 7 is the fourth term and so on.

In (iii), 100 is the first term, 95 is the second term, 90 is the third term and so on.

Also, in (i), 2\textsuperscript{nd} term (=2) = 1\textsuperscript{st} term + 1

3\textsuperscript{rd} term (=3) = 2\textsuperscript{nd} term + 1

4\textsuperscript{th} term (=4) = 3\textsuperscript{rd} term + 1

5\textsuperscript{th} term (=5) = 4\textsuperscript{th} term + 1 and so on.

i.e., each term, except the first, is obtained by adding a fixed number 1 to its preceding term.

In (ii), 2\textsuperscript{nd} term = 3 = 1\textsuperscript{st} term + 2

3\textsuperscript{rd} term = 5 = 2\textsuperscript{nd} term + 2

4\textsuperscript{th} term = 7 = 3\textsuperscript{rd} term + 2, and so on.

i.e., each term, except the first, is obtained by adding a fixed number 2 to its preceding term.

In (iii), 2\textsuperscript{nd} term = 95 = 1\textsuperscript{st} term + (-5)

3\textsuperscript{rd} term = 90 = 2\textsuperscript{nd} term + (-5)

4\textsuperscript{th} term = 85 = 3\textsuperscript{rd} term + (-5) and so on.

i.e., each term, except the first, is obtained by adding a fixed number (-5) to its preceding term.

Thus, in all these three pattern, each term, except the first, is obtained by adding a fixed number to its preceding term.

Such number patterns are known as \textbf{arithmetic sequences} or \textbf{arithmetic progressions}.
The fixed number is called the **common difference** of the progression and is usually denoted by \( d \).

Thus in (i), common difference \((d)\) is 1, In (ii), common difference is 2 and in (iii), common difference is \(-5\).

The **first term** of an arithmetic progression (AP) is usually written as \( a \).

Thus, in general,

<table>
<thead>
<tr>
<th>If ( a ) is the first term and ( d ), the common difference of an AP, then the AP can be written as</th>
<th>( a, \ a + d, \ a + 2d, \ a+3d, \ldots )</th>
</tr>
</thead>
</table>

Note that in case of an AP, it is obvious that

common difference

\[
= \text{second term} - \text{first term} \\
= \text{third term} - \text{second term} \\
= \text{fourth term} - \text{third term} \text{ and so on.}
\]

**Example 4:** Identify which of the following are arithmetic progressions. For those which are arithmetic progressions, find the common differences:

(i) 3, 6, 12, 24,……

(ii) 12, 2, –8, –18,……

(iii) \(0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots\)

(iv) \(\frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \ldots\)

(v) 1, 1.7, 2.4, 3.1,……

(vi) \(1^2, 5^2, 7^2, 73, \ldots\)
solution:

(i) 3, 6, 12, 24,…

We see that

6 – 3 = 3
12 – 6 = 6
24 – 12 = 12

Thus, it is not an AP. [As 3 ≠ 6 ≠ 12]

(ii) 12, 2, -8, -18,…….

Here, 2 – 12 = -10
-8 – 2 = -10
-18 – (-8) = -18 + 8 = -10

Thus, it is an AP.

Here, common difference (d) is -10

(iii) 0, \(\frac{1}{4}\), \(\frac{1}{2}\), \(\frac{3}{4}\)…..

We have

\[ \frac{1}{4} - 0 = \frac{1}{4} \]
\[ \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \]
\[ \frac{3}{4} - \frac{1}{2} = \frac{1}{4} \]

Thus, it is also an AP with common difference (d) = \(\frac{1}{4}\)

(iv) \(\frac{1}{7}\), \(\frac{1}{9}\), \(\frac{1}{11}\), \(\frac{1}{13}\)…..

We have
Thus, it is not an AP (Why?) [Note that denominators 7, 9, 11, 13,….. are in AP, check!!]

(v) 1, 1.7, 2.4, 3.1,….
We have,
\[ 1.7 - 1 = 0.7 \]
\[ 2.4 - 1.7 = 0.7 \]
\[ 3.1 - 2.4 = 0.7 \]
So, it is an AP with common difference \( (d) = 0.7 \)

(vi) 1\(^2\), 5\(^2\), 7\(^2\), 73,…..
\[ 1^2 = 1, \quad 5^2 = 25, \quad 7^2 = 49, \quad 73 = 73. \]
Further,
\[ 25 - 1 = 24 \]
\[ 49 - 25 = 24 \]
\[ 73 - 49 = 24 \]
So, 1\(^2\), 5\(^2\), 7\(^2\), 73,….. is an AP with common difference \( (d) = 24 \)

**Example 5:** Find the value of \( k \) for which \( k+2, 4k -6 \) and \( 3k-2 \) form three consecutive terms of an AP.

**Solution:** \( a = k+2, \quad a+d = 4k -6, \quad a+2d = 3k-2 \), where \( d \) is the common difference.

If \( k+2, 4k-6, 3k-2 \) are in AP, then

\[ d = (a+d) - a = (4k-6) - (k+2) = 3k - 8 \]  
\[ (1) \]

Also,
\[ d = (a+2d) - (a+d) = (3k -2) - (4k - 6) = 4 - k \]  
\[ (2) \]
From (1) and (2), \(3k - 8 = 4 - k\)

or, \(k = 3\).

Thus, for \(k=3\), the given three terms will form an AP.

**Example 6:** Find AP whose first term is \(-2\) and common difference is \(-5\).

**Solution:** Here \(a = -2, \ d = -5\)

So, the required AP will be

\(a, \ a+d, \ a+2d, \ldots\)

or \(-2, -2+(-5), -2+2(-5),\ldots\)

or, \(-2, -7, -12,\ldots\)

**General Term of an AP**

Recall that if \(a\) is the first term and \(d\) is the common difference, then the AP is written as 

\(a, \ a+d, \ a+2d, \ldots\)

Note that

- first term \((= a_1) = a = a + (1 - 1) \ d\)
- second term \((= a_2) = a + d = a + (2 - 1) \ d\)
- third term \((= a_3) = a + 2d = a + (3 - 1) \ d\)
- fourth term \((= a_4) = a + 3d = a + (4 - 1) \ d\)

\[\ldots\]

- tenth term \((= a_{10}) = a + 9d = a + (10 - 1) \ d\)
- 50th term \((= a_{50}) = a + 49d = a + (50 - 1) \ d\)

Observing the pattern (coefficient of \(d\) in any term is 1 less than the term number), we can write \(n\)th term \((= a_n) = a + (n - 1) \ d\)

\[a_n = a + (n - 1) \ d\]
\( a_n \) is called the **general term** of an AP.

Sometimes, the general term of an AP is also denoted by \( t_n \).

**Example 7:** Find the 10th term of the AP:

\[ 6, 2, -2, -6, \ldots \]

**Solution:** Here \( a = 6, d = 2 - 6 = -2 -2 = -6 - (-2) = -4 \)

\[ n = 10 \]

Using \( a_n = a + (n-1) d \)

\[ a_{10} = a + (10 - 1) d = 6 + (9) (-4) = -30 \]

Thus, the 10th term of the given AP is -30.

**Example 8:** Which term of the AP: 5, 2, -1, ..., is -22?

**Solution:** Here \( a = 5, d = 2 - 5 = -3 \)

Let nth term of the AP be -22

So, using \( a_n = a + (n-1) d \), we have

\[ -22 = 5 + (n - 1) (-3) \]

\[ -22 = -3n + 8 \]

\[ n = 10 \]

Thus, 10th term of the AP will be -22.

**Example 9:** If the first term of an AP is 5 and the 15th term is 33, find the common difference.

**Solution:** Here \( a = 5, n = 15 \) and \( a_{15} = 33 \)

Using \( a_n = a + (n-1) d \), we have

\[ 33 = 5 + (15 - 1) d \]

or, \( 14d = 28 \)

\[ d = 2 \]

Hence, the common difference of the AP is 2.
Example 10: Find the first term of an AP whose 14th term is −47 and common difference is −4.

Solution: Let the first term of the AP be a.

Here, \( d = -4, \ a_{14} = -47. \)

Using \( a_n = a + (n-1)d, \) we have

\[
\begin{align*}
a_{14} &= a + (14 - 1)(-4) = -47 \\
a &= -47 + 52 = 5
\end{align*}
\]

Hence, the first term of the AP is 5.

Example 11: Find the AP whose 7th term is 25 and 15th term is 41.

Solution: We are given that \( a_7 = 25 \) and \( a_{15} = 41. \)

Let the AP be \( a, a+d, a+2d, \ldots. \)

Now, \( a_7 = 25 = a + (7 - 1) \ d \)

or \( a + 6d = 25 \) \hspace{2cm} (1)

Similarly \( a_{15} = 41 = a + (15 - 1) \ d \)

or, \( a+14d = 41 \) \hspace{2cm} (2)

Solving (1) and (2), \( a = 13 \) and \( d = 2 \)

Thus, the required AP is 13, 15, 17,.....

Example 12: Check whether

(i) 185 is a term of the AP : −40, −15, 10,.....
(ii) −27 is a term of the AP : 5, 2, -1,.....

Solution: (i) Here \( a = -40, \ d = -15 - (-40) = 25 \)

Let 185 be the \( n \)th term of the AP.

So, using \( a_n = a + (n - 1) \ d, \) we have

\[ 185 = -40 + (n -1) 25 \]
or, \( 185 = -40 + 25n - 25 \)

or, \( 25n = 250 \)

or, \( n = 10 \)

As \( n = 10 \), is a natural number, therefore, 185 can be a term of the given AP. (in fact it is 10\(^{th}\) term check!).

(ii) Here \( a = 5, d = 2 - 5 = -3 \).

Let \(-27\) be nth term of the AP.

Then \[-27 = 5 + (n-1)(-3)\]

\[-27 = 5 - 3n + 3\]

\[3n = 35\]

\[n = \frac{35}{3}, \text{ which is not a natural numbers.}\]

So, \(-27\) cannot be a term of given AP.

**Example 13:** Find the 12\(^{th}\) term from the last term (towards the first term) of the AP 9, 6, 3, ……, -63.

**Solution:** Here, \( a = -63 \)

d will be the negative of common difference of AP. \( = - (6 - 9) = 3 \)

Using, \( a_{12} = a + (n - 1) \cdot d \)

\[= -63 + (12 - 1) \cdot 3\]

\[= -63 + 33\]

\[= -30\]

Hence, 12\(^{th}\) term from the last term of the given AP will be \(-30\).
Example 14: Show that $a_1, a_2, \ldots, a_n$ form an AP where $a_n$ is defined as $a_n = 2 + 3n$.

Solution: Putting $n=1$ in $a_n$, we get

$$a_1 = 2+3(1) = 5$$

Similarly,

$$a_2 = 2+3(2) = 8$$

$$a_3 = 2 + 3 (3) = 11$$

$$a_4 = 2 + 3 (4) = 14$$, and so on

Clearly, 5, 8, 11, 14,…. form an AP, as common difference $d = 8−5 = 11−8 = 14−11 = 3$

If nth term $a_n$ of a number pattern is of the term $x+ny$ where $x$ and $y$ are real numbers and $n$ is a natural number, then $a_1, a_2, a_3, \ldots$ forms an AP.

3. Geometric Progression

Let us revisit Example 2 [pattern (iii)] and Example 3 [pattern (ii)]. We reproduce them as follows:

(i) 1, 2, 4, 8, ……..

(ii) 54, 18, 6, 2, $\frac{2}{3}$, …..

Certainly, (i) and (ii) are not arithmetic progressions (check!!)

We have seen that each term in (i), except the first, is obtained by multiplying its preceding term by a fixed number 2. [2 = 1x2, 4 = 2x2, 8 = 4x2, …]

In (ii), each term, except the first, is obtained by multiplying its preceding term by a fixed number $\frac{1}{3}$ [18 = $54 \times \frac{1}{3}$, 6 = $18 \times \frac{1}{3}$, 2 = $6 \times \frac{1}{3}$, …..]

In both the cases, each term, except the first, is obtained by multiplying its preceding term by a fixed number.

Such number patterns are known as geometric sequences or geometric progressions.
The fixed number is called the **common ratio** of the progression and is usually denoted by \( r \). Thus in (i), common ratio \( r \) is 2 and in (ii) common ratio \( r \) is \( \frac{1}{3} \).

The first term of a geometric progression (GP) is usually written as \( a \) (as in case of AP). Thus, in general.

**If \( a \) is the first term and \( r \), the common ratio of a GP, then GP can be written as \( a, ar, ar^2, ar^3, \ldots \).**

Note that in case of a GP,

**Common ratio**

\[
= \frac{\text{second term}}{\text{first term}}
\]

\[
= \frac{\text{third term}}{\text{second term}}
\]

\[
= \frac{\text{fourth term}}{\text{third term}}
\]

and so on.

**Example 15:** Identify which of the following are geometric progressions. For those which are geometric progressions, find the common ratios:

(i) 3, 6, 12, 24, …

(ii) \( \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots \)

(iii) –2, –6, –18, –54

(iv) \( \frac{4}{3}, \frac{8}{5}, \frac{16}{7}, \frac{32}{9}, \ldots \)

**Solution:**

(i) 3, 6, 12, 24, …

Here \( a = 3 \). We have
So, it is a GP. Its common ratio \( r = 2 \).

(ii) \( \frac{3}{2}, \frac{3}{4}, \frac{3}{16}, \frac{3}{32} \)..........

Here \( a = \frac{3}{2} \)

\[
\frac{3}{2} \div \frac{3}{4} = \frac{3}{2} \times \frac{4}{3} = \frac{1}{2} \quad \left[ \frac{1}{2} \neq \frac{1}{4} \right]
\]

So, it is not a GP.

(iii) \( \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81} \).............

Here \( a = -\frac{1}{3} \)

\[
\frac{1}{3} \div \left( -\frac{1}{3} \right) = \frac{1}{9} \times \frac{3}{1} = -\frac{1}{3},
\]

\[
-\frac{1}{3} \div \frac{1}{9} = -\frac{1}{27} \times \frac{9}{1} = -\frac{1}{3}
\]

\[
\frac{1}{81} \div -\frac{1}{27} = \frac{1}{81} \times \frac{27}{1} = -\frac{1}{3}, \ldots...
\]

So, it is a GP. Its common ratio \( r \) is \( -\frac{1}{3} \).

(iv) \(-2, -6, -18, -54, \ldots\)
Here \( a = -2 \)

\(-6 \div (-2) = 3\)

\(-18 \div (-6) = 3\)

\(-54 \div (-18) = 3\)

So, it is a GP with common ratio \( r = 3 \).

\[ \frac{4}{3}, \frac{8}{5}, \frac{16}{7}, \frac{32}{9}, \ldots \]

Here \( a = \frac{4}{3} \)

\[ \frac{8}{5} \div \frac{4}{3} = \frac{8}{5} \times \frac{3}{4} = \frac{6}{5} \ldots \]

\[ \frac{16}{7} \div \frac{8}{5} = \frac{16}{7} \times \frac{5}{9} = \frac{10}{7} \]

So, it is not a GP as \( \frac{6}{5} \neq \frac{10}{7} \).

From the numerators, it appears as the numbers are in GP but in fact they are not.

**Example 16:** Find the value of \( k \) for which \( 4k + 4, 6k - 2, 9k - 13 \) form a GP.

**Solution:** Since \( 4k + 4, 6k - 2 \) and \( 9k - 13 \) form a GP,

\[ \frac{6k - 2}{4k + 4} = \frac{9k - 13}{6k - 2} \quad (= r) \]

\[ (6k - 2)(6k - 2) = (4k + 4)(9k - 13) \]

\[ 36k^2 - 24k + 4 = 36k^2 - 52k + 36k - 52 \]

\[ = 36k^2 - 16k - 52 \]

\[ 8k = 56 \]

\[ k = 7 \]
Example 17: Find a GP whose first term is $2\sqrt{2}$ and common ratio is $\frac{1}{\sqrt{2}}$.

Solution: Here $a = 2\sqrt{2}$, $r = \frac{1}{\sqrt{2}}$

So, the required GP will be

$a, ar, ar^2, \ldots$

i.e., $2\sqrt{2}, 2\sqrt{2} \times \frac{1}{\sqrt{2}}, 2\sqrt{2} \times \left(\frac{1}{\sqrt{2}}\right)^2, \ldots$

or, $2\sqrt{2}, 2, \sqrt{2}, \ldots$

• General Term of a GP

Recall that if $a$ is the first term and $r$ is the common ratio, then the GP is written as $a, ar, ar^2, \ldots$

Note that

- first term $(= a_1) = a = ar^{1-1}$
- second term $(= a_2) = ar = ar^{2-1}$
- third term $(= a_3) = ar^2 = ar^{3-1}$
- fourth term $(= a_4) = ar^3 = ar^{4-1}$

......

......

- tenth term $(= a_{10}) = ar^9 = ar^{10-1}$

Observing the pattern (exponent of $r$ in any term is 1 less than the term number,) We can write

- $n$th term $(= a_n) = ar^{n-1}$

\[ a_n = ar^{n-1} \]

$a_n$ is called the general term of a GP.

Sometimes, the general term of a GP is also denoted by $t_n$. 

Example 18: Find 8th term of the GP:

\[27, -18, 12, -18, \ldots\]

Solution: Here \(a = 27, \quad r = \frac{-18}{27} = \frac{12}{-18} = \frac{-18}{12} = -\frac{2}{3}\)

Using \(a_n = ar^{n-1}\),

\[a_8 = (27) \left(-\frac{2}{3}\right)^{8-1} = (27) \left(-\frac{2}{3}\right)^7\]

\[= 27 \left(-\frac{128}{2187}\right) = -\frac{128}{81}\]

Hence, 8th term of the given GP is \(-\frac{128}{81}\).

Example 19: Which term of the GP:

\[36, -12, 4, \ldots\]

is \(\frac{4}{81}\)?

Solution: Here, \(a = 36, \quad r = \frac{-12}{36} = \frac{4}{-12} = -\frac{1}{3}\)

Let nth term of the G.P. be \(\frac{4}{81}\)

Using \(a_n = ar^{n-1}\), we have:

\[\frac{4}{81} = 36 \left(-\frac{1}{3}\right)^{n-1}\]

or,

\[\frac{4}{81 \times 36} = \left(-\frac{1}{3}\right)^{n-1}\]

or,

\[\frac{1}{81 \times 9} = \left(-\frac{1}{3}\right)^{n-1}\]

\[\left(-\frac{1}{3}\right)^6 = \left(-\frac{1}{3}\right)^{n-1}\]

or, \(n-1 = 6\)

or, \(n = 6 + 1 = 7\)

Thus, \(\frac{4}{81}\) is the seventh term of the GP.
Example 20: If the first term of a GP is $\sqrt{3}$ and its 10th term is $16\sqrt{6}$, find the common ratio of the GP.

Solution: Here, $a = \sqrt{3}$

Let $r$ be the common ratio.

\[
a_{10} = 16\sqrt{6}
\]

i.e., \[ar^9 = 16\sqrt{6}\]

or, \[\sqrt{3} \cdot r^9 = 16\sqrt{6}\]

or, \[r^9 = \frac{16\sqrt{6}}{\sqrt{3}} = 16\sqrt{2}\]

or, \[r^n = \sqrt{512} = (\sqrt{2})^9\]

so, \[r = \sqrt{2}\]

Thus, the required common ratio is $\sqrt{2}$

Example 21: Find the first term of a GP whose 10th term is $\frac{3}{1024}$ and whose common ratio is $\frac{1}{2}$.

Solution: Here, \[r = \frac{1}{2}\]

and \[a_{10} = \frac{3}{1024}\]

Let the first term be $a$.

So, \[ar^9 = \frac{3}{1024}\]

or, \[a \left(\frac{1}{2}\right)^9 = \frac{3}{1024}\]
or, \[ \frac{a}{512} = \frac{3}{1024} \]

or, \[ a = \frac{3 \times 512}{1024} = \frac{3}{2} \]

i.e., the first term of the GP is \( \frac{3}{2} \).

**Example 22:** Second term of a GP is 3 and its fifth term is \( \frac{81}{8} \). Find its eight term.

**Solution:** Let the G.P. be \( a, ar, ar^2, \ldots \). 

Now, \( a_2 = ar = 3 \) \hspace{1cm} (1) 

and \( a_5 = ar^4 = \frac{81}{8} \) \hspace{1cm} (2)

Dividing (2) by (1), we have:

\[ \frac{ar^4}{ar} = \frac{\frac{81}{8}}{3} \]

or \[ r^3 = \frac{27}{8} = \left(\frac{3}{2}\right)^3 \]

or \[ r = \frac{3}{2} \]

Putting this value in (1),

\[ a \times \frac{3}{2} = 3 \]

so, \( a = 2 \)

So, 8th term = \( ar^7 \)

\[ = 2 \left(\frac{3}{2}\right)^7 \]
(4) **Sum of first \( n \) terms of an AP**

Let the first \( n \) terms of an AP be 

\[ a, a+d, a+2d, a+3d, \ldots, [a+(n-2)d], [a+(n-1)d] \]

Let 

\[ S_n = a + (a+d) + (a+2d) + \ldots + [a+(n-2)d] + [a+(n-1)d] \]  \hspace{1cm} (1) 

Also, \( S_n = [a+(n-1)d] + [a+(n-2)d] + [a+(n-3)d] + \ldots + (a+d) + a \)  \hspace{1cm} (2)

[Writing the terms in a reverse order]

Adding (1) and (2), term by term.

\[ 2S_n = [2a + (n-1)d] + [2a + (n-1)d] + \ldots + [2a + (n-1)d] \]

\[ n \text{ times} \]

Note:

\[
\begin{align*}
a + [a + (n-1) \times d] &= 2a + (n-1) \times d \\
(a+d) + [a + (n-2) \times d] &= 2a + (n-1) \times d \\
(a+2d) + [a + (n-3) \times d] &= 2a + (n-1) \times d \\
[a + (n-2) \times d] + a+d] &= 2a + (n-1)d \\
[a + (n-1) \times d] + a &= 2a + (n-1) \times d
\end{align*}
\]

\[ 2S_n = n [2a + (n-1) \times d] \]

**So,** 

\[ S_n = \frac{n}{2} [2a + (n-1) \times d] \]
Thus, the sum of first \( n \) terms of an AP is given by

\[ S_n = \frac{n}{2} [2a + (n-1) d] \]

Note that

last term of the AP = \( \alpha_n = a + (n-1) d \).

Let \( l = a + (n-1) d \).

So,

\[ S_n = \frac{n}{2} [2a + (n-1) d] = \frac{n}{2} [a + a + (n-1) d] = \frac{n}{2} [a + l] \]

Hence \( S_n = \frac{n}{2} [a+l] \),

where \( l \) is the last term or \( n \)th term.

The above method is similar to the one used by the great German mathematician Carl friedrich Gauss (1777-1855) when he was in elementary school. His teacher asked the class to find the sum of first 100 natural numbers. Gauss found the sum as follows:

\[
\begin{align*}
1 + 2 + 3 + 4 + \ldots + 98 + 99 + 100 \\
100 + 99 + 98 + 97 + \ldots + 3 + 2 + 1 \\
\hline
101 + 101 + 101 + 101 + \ldots + 101 + 101 + 101
\end{align*}
\]

101, 100 times

i.e., sum = \( \frac{101 \times 100}{2} = 5050 \)
Example 23: find the sum of first

(i) 100 natural numbers.
(ii) 1000 natural numbers.

Solution:

(i) Here $a=1, \ d=1, \ n = 100$

Thus, $S_{100} = \frac{100}{2} [2 \times 1 + (100-1) \times 1] = 50 \times 101 = 5050$

Alternatively, we may use the formula

$$S_n = \frac{n(a + l)}{2}$$

$$S_{100} = \frac{100}{2} (1+100) = 5050$$

(ii) Using the formula for $S_n$,

$$S_{1000} = \frac{1000}{2} [2 \times 1 + (1000-1)1]$$

$$= 500 \times 1001$$

$$= 500500$$

Also by using the formula:

$$S_n = \frac{n}{2} (a + l)$$

$$S_{1000} = \frac{1000}{2} (1 + 1000)$$

$$= 500 \times 1001$$

$$= 500500$$

Example 24: Find the sum of first 50 terms of the AP; 1, 3, 5, 7, ...........

Solution: Here $a=1, \ d=2, \ n=50$

using the formula
\[ S_n = \frac{n}{2} [2a + (n - 1)d] \]

\[ S_{50} = \frac{50}{2} [2 \times 1 + (50 - 1)(2)] \]

\[ = 25 [2 + 98] \]

\[ = (25) \times (100) = 2500 \]

Refer to geometric pattern given in “Introduction” under (iv). Here
\[ 1 + 3 + 5 + \ldots + 50 = 2500 = (50)^2 \]

Using this formula, now you can find this sum to \( n \) terms and the sum
\[ 1 + 3 + 5 + \ldots \text{upto} \ n \ \text{terms} = n^2 \]

**Example 25:** How many terms of the AP : 1, 4, 7, .... are needed to give the sum 715?

**Solution:** Here \( a = 1 \), \( d = 4 - 1 = 3 \) and \( S_n = 715 \).

We have to find \( n \).

Using the formula

\[ S_n = \frac{n}{2} [2a + (n - 1)d], \]

we have

\[ 715 = \frac{n}{2} [2 \times 1 + (n - 1) \times 3], \]

\[ = \frac{n}{2} [2 + 3n - 3], \]

\[ = \frac{n}{2} [3n - 1], \]

or, \( 3n^2 - n - 1430 = 0 \)

This is a quadratic equation. Using quadratic formula,

\[ n = \frac{1 \pm \sqrt{1 + 4(3)(1430)}}{6} = \frac{1 \pm \sqrt{17161}}{6} \]
So, \( n = \frac{1 \pm 131}{6} \) or \( 22, -\frac{65}{3} \).

Certainly \( n \) cannot be \( -\frac{65}{3} \).

Thus, \( n = 22 \),
i.e., the required number of terms is 22.

**Example 26:** Find the sum of first 20 terms of the AP: 3, 1, –1, -3, ……

**Solution:** Here \( a=3, \ d= 1–3 = -2, \ n = 20. \)

Using \( S_n = \frac{n}{2} [2a + (n-1)d] \), we get

\[
S_{20} = \frac{20}{2} [2 \times 3 + (20-1)(-2)]
= 10 (6 - 38) = -320.
\]

Thus, the sum of first 20 terms of the given AP is -320.

**Example 27:** The sum of first 25 terms of an AP is 1600. If its common difference is 5, find the first term.

**Solution:** Here \( d=5, \ n=25 \) and \( S_{25} = 1600 \)

Using \( S_n = \frac{n}{2} [2a + (n-1) d] \), we get

\[
1600 = \frac{25}{2} [2a + (25-1)(5)]
\]

or \( 1600 = \frac{25}{2} [2a + 120] \)

or \( 3200 = 50a + 3000 \)

or \( a = 4 \)

Thus, first term of the AP is 4.
Example 28: The sum of first 22 terms of an AP is 715. If its first term is 1, find the common difference of the AP.

Solution: Here, \(a = 1\), \(n = 22\) and \(S_{22} = 715\).

Using the formula.

\[
S_n = \frac{n}{2} [2a + (n-1)d],
\]

we have

\[
715 = \frac{22}{2} [2 \times 1 + 21d]
\]

or \(715 = 11[2 + 21d]\)

or \(2 + 21d = \frac{715}{11} = 65\)

or \(21d = 65 - 2 = 63\)

So,

\[
d = \frac{63}{21} = 3
\]

Thus, the required common difference is 3.

Example 29: The sum of first \(n\) term of an AP is \(3n^2 - 5n\). Find its 10th term.

Solution: \(S_n = 3n^2 - 5n\)

Note that \(a_n = S_n - S_{n-1}\)

So,

\[
a_{10} = S_{10} - S_9
\]

\[
= [3(10)^2 - 5x(10)] - [3(9)^2 - 5x(9)]
\]

\[
= [300 - 50] - [243 - 45]
\]

\[
= (250) - (198) = 52
\]

Thus, the required 10th term is 52.
Student’s Support Material
Warm up (W1) - Recognising Number Patterns

Which of the following are showing a number pattern? Observe the number patterns and write the next three terms.

1, 3, 5, 7, ...
4, 6, 8, ...
-7, -6, -4, -2, 0 ...
4, 9, 16, 25, ...
10, 30, 20, 60, 30, 90, ...
14, 16, 18, ...
-100, 0, -100, 0, -100, 0, ...
2, 7, 12, 17, ...
3, 5, 6, 10, 9, 15, ...
### Self Assessment Rubric 1- Warm Up (W1)

<table>
<thead>
<tr>
<th>Parameters of assessment</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Able to observe number pattern (if any)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to generate the next terms using pattern</td>
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<td></td>
<td></td>
<td></td>
</tr>
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</table>
Observe the pattern in the following geometrical design. Write your observation.

Make design of your choice using patterns on the following sheet.
### Self Assessment Rubric 2- Warm Up (W2)

#### Parameters of assessment

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<tr>
<th>Parameters of assessment</th>
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<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to observe pattern in shapes (if any)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to generate the next in turn using pattern</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Name of Student _____________________    Date___________

Observe the following diagrams and write your observations. Do you observe any pattern? Draw the next shape for each of the following. Express the given designs in terms of numbers.

<table>
<thead>
<tr>
<th>Diagrams</th>
<th>Number of squares used in each step of the pattern sequence</th>
<th>Rule used for making the next in turn</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
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<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram 3" /></td>
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</tr>
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</table>
### Self Assessment Rubric 3- Warm Up (W3)

#### Parameters of assessment

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</tr>
</thead>
<tbody>
<tr>
<td>Able to observe pattern in shapes (if any)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to generate the next in turn using pattern</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to associate patterns and numbers</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Student’s Worksheet 4

Pre Content (P1) – Match stick Activity

Name of Student _____________________    Date___________

Using matchsticks create row of squares as shown below and count the number of matchsticks required in each case.

- Row containing 1 square
- Row containing 2 squares
- Row containing 3 squares

Complete the following table:

<table>
<thead>
<tr>
<th>Row containing squares</th>
<th>No. of matchsticks required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

Q1. What do you observe about the number of matchsticks required with addition of one square in each case?

Q2. How many matchsticks will be required for row containing 10 squares, 20 squares, 50 squares?
Self Assessment Rubric 4- Pre Content (P1)

### Parameters of assessment

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<tr>
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<th>No Understanding</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to make a sequence of numbers using patterns</td>
<td>![Green Square]</td>
<td>![Green Square]</td>
<td>![Green Square]</td>
<td>![Green Square]</td>
</tr>
<tr>
<td>Able to generate the specified term using the pattern</td>
<td>![Green Square]</td>
<td>![Green Square]</td>
<td>![Green Square]</td>
<td>![Green Square]</td>
</tr>
</tbody>
</table>
Student’s Worksheet 5
Pre Content (P2) – Visualising Specified Terms

Name of Student _____________________    Date___________

Observe the following patterns. Count the number of squares in each term. Guess the no. of squares required for mentioned term.

1. 

2. 

3. 

4. Observe the number sequence and complete the next three terms of the sequence:
   i. 2, 7, 12, 17, _, _, _
   ii. 5, 18, 31, 44, _, _, _
   iii. 11, 22, 33, 44, _, _, _
### Self Assessment Rubric 5 - Pre Content (P2)

#### Parameters of assessment

<table>
<thead>
<tr>
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<th>□</th>
<th>□</th>
<th>□</th>
<th>□</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to write the next terms in the given sequence of numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to draw the specified shape using the given shapes by observing pattern</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Let $a$ be the first term, $d$ be the common difference of a given sequence. Observe the following diagram and write the successive terms in terms of $a$ and $d$.

First term, $a_1 = .........$

Second term, $a_2 = ........$

Third term, $a_3 = ........$

Third term, $a_4 = ........$

Third term, $a_5 = ........$

What do you think would be the 18$^{th}$ term?

__________________________________________________________________________________

Write the formula for $n$th term.

__________________________________________________________________________________
If a is the first term of an A.P. and d is the common difference then the general term of the sequence will be written as

________________________________________________________________________

________________________________________________________________________

2. Brainstorming

Ragini arranged rectangular bars in ascending order as shown below. What do you think would be the height of the 6th bar? How will you find the height of 20th bar?

3. Find the 6th term of A.P 14, 17, 20, ...
### Parameters of assessment

<table>
<thead>
<tr>
<th>Parameters of assessment</th>
<th>No Understanding</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to write the formula for nth term of an A.P.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to write a general A.P.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Student’s Worksheet 7
Content Worksheet (CW2) – Recognising an A.P.

Name of Student _____________________    Date___________

1. Check which of the following sequences is an A.P. Justify your answer.
   a)  12, 14, 16, 18, ......

   Here, \( a_1 = \ldots \ldots \ldots \)
   \[ a_2 = \ldots \ldots \ldots \]
   \[ a_2 - a_1 = \ldots \ldots \ldots \]
   \[ a_3 = \ldots \ldots \ldots \]
   \[ a_3 - a_2 = \ldots \ldots \ldots \]
   Is \( a_2 - a_1 = a_3 - a_2 \)?
   What do you conclude?

   Here, \( a_1 = \ldots \ldots \ldots \)
   \[ a_2 - a_1 = \ldots \ldots \ldots \]
   \[ a_2 = \ldots \ldots \ldots \]
   \[ a_3 = \ldots \ldots \ldots \]
   \[ a_3 - a_2 = \ldots \ldots \ldots \]
   Is \( a_2 - a_1 = a_3 - a_2 \)?
   What do you conclude?

   Here, \( a_1 = \ldots \ldots \ldots \)
   \[ a_2 - a_1 = \ldots \ldots \ldots \]
   \[ a_2 = \ldots \ldots \ldots \]
   \[ a_3 = \ldots \ldots \ldots \]
   \[ a_3 - a_2 = \ldots \ldots \ldots \]
   Is \( a_2 - a_1 = a_3 - a_2 \)?
   What do you conclude?

   Here, \( a_1 = \ldots \ldots \ldots \)
   \[ a_2 - a_1 = \ldots \ldots \ldots \]
   \[ a_2 = \ldots \ldots \ldots \]
   \[ a_3 = \ldots \ldots \ldots \]
   \[ a_3 - a_2 = \ldots \ldots \ldots \]
   Is \( a_2 - a_1 = a_3 - a_2 \)?
   What do you conclude?

   Here, \( a_1 = \ldots \ldots \ldots \)
   \[ a_2 - a_1 = \ldots \ldots \ldots \]
   \[ a_2 = \ldots \ldots \ldots \]
   \[ a_3 = \ldots \ldots \ldots \]
   \[ a_3 - a_2 = \ldots \ldots \ldots \]
   Is \( a_2 - a_1 = a_3 - a_2 \)?
   What do you conclude?

b)  -4, -7, -10, -13, ......

   Here, \( a_1 = \ldots \ldots \ldots \)
   \[ a_2 = \ldots \ldots \ldots \]
   \[ a_2 - a_1 = \ldots \ldots \ldots \]
   \[ a_3 = \ldots \ldots \ldots \]
   \[ a_3 - a_2 = \ldots \ldots \ldots \]
   Is \( a_2 - a_1 = a_3 - a_2 \)?
   What do you conclude?

   Here, \( a_1 = \ldots \ldots \ldots \)
   \[ a_2 - a_1 = \ldots \ldots \ldots \]
   \[ d = \ldots \ldots \ldots \]
   \[ a_n = a_1 + (n-1)d \]
   \[ = \]
   \[ = \]
   \[ a_{n-1} = \]
   \[ = \]
   Is \( a_n - a_{n-1} \) free from \( n \)?
   What do you conclude?

   Here, \( a_1 = \ldots \ldots \ldots \)
   \[ a_2 - a_1 = \ldots \ldots \ldots \]
   \[ d = \ldots \ldots \ldots \]
   \[ a_n = a_1 + (n-1)d \]
   \[ = \]
   \[ = \]
   \[ a_{n-1} = \]
   \[ = \]
   Is \( a_n - a_{n-1} \) free from \( n \)?
   What do you conclude?
2. Investigation: For which of the following sequences the common difference will be same. Justify your answer
   a) \( a_n = 1 + 2n \)
   b) \( a_n = -1 + 2n \)
   c) \( a_n = 1 + 2n^2 \)
   d) \( a_n = n^2 + 2n \)

3. Write the first term and common difference of following A.P’s.
   a) \(-4, -2, 0, 2, \ldots\)
   b) \(0.7, 1.0, 1.3, 1.6, \ldots\)
### Self Assessment Rubric 7

#### Content Worksheet (CW2)

<table>
<thead>
<tr>
<th>Parameters of assessment</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to tell whether a given sequence is an A.P. or not</td>
<td>A</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Able to write the nth term of given sequence.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to predict about the common difference, given the nth term.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Student’s Worksheet 8

Content Worksheet (CW3) – Terms of an A.P

Name of Student _____________________    Date___________

1. Write the first four terms of the following AP’s whose nth term is given.
   a) \( a_n = 2n + 1 \)
   
   \[
   \begin{array}{cccc}
   a_1 = & a_2 = & a_3 = & a_4 = \\
   \end{array}
   \]
   b) \( a_n = 3n - 1 \)
   
   \[
   \begin{array}{cccc}
   a_1 = & a_2 = & a_3 = & a_4 = \\
   \end{array}
   \]
   c) \( a_n = 7 - 6n \)
   
   \[
   \begin{array}{cccc}
   a_1 = & a_2 = & a_3 = & a_4 = \\
   \end{array}
   \]
   d) \( a_n = 3n \)
   
   \[
   \begin{array}{cccc}
   a_1 = & a_2 = & a_3 = & a_4 = \\
   \end{array}
   \]

2. Write the specified term of the given A.P. whose nth term is given
   a) \( a_n = 2n + 1 \)
   
   \[
   a_{20} = \\
   \]
   b) \( a_n = 3n - 1 \)
   
   \[
   a_{100} = \\ a_{120} = \\
   \]
   c) \( a_n = 7 - 6n \)
   
   \[
   a_1 = \\ a_{22} = \\
   \]
   d) \( a_n = 3n \)
   
   \[
   a_{15} = \\ a_{25} = \\
   \]
3. Rahul said, “If the nth term of a sequence is linear then it must form an A.P.”
What do you say? Justify your answer with an example.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

4. Write your comments.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

5. If the nth term of an A.P. is 3n+7 then which of the following cannot be the terms of the sequence. Justify your answer.

6. The nth term of a sequence is 3n^2 + 5. Will the sequence be an A.P? Why? Why not?

1. Can 6n+1 be the common difference of A.P?
2. Which of the following expressions will form an A.P. Justify your answer.
   a) $a_n = 5 - 3n$
   b) $a_n = \frac{5}{3n}$
   c) $a_n = 5 + 3n$
   d) $a_n = \frac{1}{5+3n}$
   e) $a_n = 5(3n)$
   f) $a_n = \sqrt{5}(3n)$

3. Complete the following table.

<table>
<thead>
<tr>
<th>First term</th>
<th>Common difference</th>
<th>First five terms of A.P.</th>
<th>$a_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{7}{2}$</td>
<td>$\frac{1}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sqrt{3}$</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-4</td>
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Self Assessment Rubric 8
Content Worksheet (CW3)

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<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to write the specified term when nth term of an A.P. is given</td>
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<td></td>
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<tr>
<td>Able to understand that a linear expressions in forms an A.P.</td>
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<td></td>
<td></td>
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<tr>
<td>Able to understand that common difference of an A.P is free from n.</td>
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</tbody>
</table>
1. How do you calculate any specified term of an A.P.? What do you need for calculating it?
   Let $a_1$ is the first term and $d$ is the common difference.
2. Show that the sequence 15, 30, 45, 60, ..... is an A.P. Find its 100th term.

3. If you are given the first term and the common difference of an A.P. then explore the method of finding the 20th term. Explain by taking any example.

4. The first term of an A.P. is 4 and common difference is 5. The last term is 254. Write the sequence. Investigate the method of finding 9th term from the end.
5. Given a sequence 6, 11, 16, 21, ...... Is it an A.P.? Can 112 be a term of this sequence? Explain your answer with justification.

6. How many terms are there in the sequence of multiples of 7 from 21 to 343?

7. The 10th term of an A.P. is 52 and the 17th term is 20 more than the 13th term. Can you find the A.P.? Explain your answer.
8. Two A.P.'s have the same common difference. The common difference between their 50\textsuperscript{th} terms is 1089. What is the common difference between their 500\textsuperscript{th} terms?

9. Which term of the A.P. 90, 88, 86, 84, .... is 0? Write the next four terms after 0.

10. Which term of the A.P. -18, -16, -14, .... is 0? Write the next four terms after 0.
11. Can 0 be a term of the A.P. -155, -151, -147...? Why? Why not?

12. Can 0 be a term of the A.P. 66, 62, 58...? Why? Why not?
Self Assessment Rubric 9
Content Worksheet (CW4)

<table>
<thead>
<tr>
<th>Parameters of assessment</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Able to find the specified term of an A.P when A.P. is given</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to find whether a given term belongs to the given A.P.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to find the given term is which term of the given A.P.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Hands on Activity - To check whether the given sequence is an A.P. or not

**Aim:** By paper cutting and pasting check whether the given sequence is an A.P. or not

- sequence I : 4, 7, 10, 13, ...
- sequence II : 1, 3, 4, 6, ...

**Material required:**
- Coloured paper, a pair of scissors, glue, ruler, sketch pen

**Procedure**

step 1 Consider the given sequence.
Step 2 Cut rectangular strips of dimensions 4cmX 2cm, 7cmX 2cm, 10cmX 2cm, 13cmX 2cm. of different colours.

Step 3 Draw a straight line on a paper.

Step 4 Paste the rectangular strip of dimension 4cmX 2cm on the line such that the base of the strip touches the line.

Step 5 Now take the rectangular strip of dimension 7cmX2cm and paste it adjacent to the previous strip without leaving any gap.
**Step 6** Take the strip of dimension 10cmX2cm and paste it adjacent to the previous strip without leaving any gap.

**Step 7** Similarly take the strip of dimension 13cmX2cm and paste it adjacent to the previous one as shown below.

**Step 8** Take a thread and check the difference of heights between two consecutive rectangular strips.

**Step 9** Write the observations.
Repeat the activity for the second sequence and write the result.

Answer the following questions:

1. Write a sequence which is an A.P.
2. What is the common difference of the sequence -2, 0, 2, 4, 6, ……
3. Is 3, 3, 3, 3, …………… an A.P.?
4. What is the common difference of a sequence of multiples of 100?
5. Is the sequence of odd natural numbers an A.P.?
6. Check whether sequence having nth term 2n+3 will be an A.P. or not?
7. Write a sequence having common difference 7.
8. Check whether the given sequence is an A.P. or not. If it is an A.P. find the common difference
   i) 10, 10, 10, 10……
   ii) 1, -1, 1, -1, …
### Self Assessment Rubric 10
#### Content Worksheet  (CW5)

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<tr>
<th>Parameters of assessment</th>
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<tbody>
<tr>
<td>Able to visualise whether a given sequence is an A.P. or not</td>
<td>✅</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to solve problems based on A.P.</td>
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</tbody>
</table>
1. Write your comments about

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2. Given below are some sequences. Recognise G.P.’s. Write the first term and the common ratio.
   a) 2, 2^2, 2^3, 2^4, 2^5 ...
   b) \(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \ldots\)
   c) 4, 9, 16, 25, 36 ...
   d) 1, 2, 3, 4, 5 ...
   e) 5, 15, 45...
3. Take the suitable value for first term, select a common ratio. Write 5 G.P’s.
## Self Assessment Rubric 11

**Content Worksheet (CW6)**

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<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
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</thead>
<tbody>
<tr>
<td>Able to tell whether a given sequence is a G.P. or not</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Able to write a G.P.</td>
<td></td>
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</tr>
</tbody>
</table>
1. Write the formula for finding the general term of a G.P.
   First term = a
   Second term = ar
   Third term = ar^2
   .
   .
   .
   nth term =

2. Write a G.P. whose first term is 3 and common ratio is 4. How did you proceed?

3. Write the general term of a G.P whose first term is -1 and common ratio is -1.
## Self Assessment Rubric 12

### Content Worksheet (CW7)

<table>
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<tr>
<th>Parameters of assessment</th>
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</thead>
<tbody>
<tr>
<td>Able to justify whether a given sequence is an A.P. or a G.P.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to write the general term of a G.P.</td>
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</tbody>
</table>
Solve the following.

1. Consider the sequence 5, 15, 45,… Is it a G.P. Find its 8th term.

2. Find the 19th term of \(\frac{1}{2}, -\frac{1}{3}, \frac{2}{9}, \ldots\)
Find the $n$th term of
$9, -6, 4, \ldots$.

Find the $n$th term of
$72, -18, -9/2, \ldots$.
5. Find a G.P. whose 3rd and 6th terms are 1 and $-\frac{1}{8}$ respectively. Write down its 10th term also.

6. The 3rd term of a G.P. is $\frac{2}{3}$ and the 6th term is $\frac{2}{81}$, find the 8th term.

7. Which term of sequence 1, 2, 4, 8 ... is 256?
Self Assessment Rubric 13
Content Worksheet (CW8)

<table>
<thead>
<tr>
<th>Parameters of assessment</th>
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</thead>
<tbody>
<tr>
<td>Able to apply the formula for general term of a G.P and find the unknown variable</td>
</tr>
</tbody>
</table>

- **No Understanding**
- Understanding of concept but not able to apply
- Understanding of concept, can apply but commit errors in calculation
- Understanding of concept, can apply accurately
1. Observe the given pattern and find the formula for sum of first n terms of an A.P.

<table>
<thead>
<tr>
<th>Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1 = a_1$</td>
</tr>
<tr>
<td>$S_2 = a_1 + (a_1 + d) = 2a_1 + d$</td>
</tr>
<tr>
<td>$S_2 = a_1 + a_2$</td>
</tr>
<tr>
<td>= $2/2[a_1 + a_2]$</td>
</tr>
<tr>
<td>$S_2 = 2a_1 + d = 2/2[a_1 + a_2]$</td>
</tr>
<tr>
<td>$S_3 = a_1 + (a_1 + d) + (a_1 + 2d) = 3a_1 + 3d$</td>
</tr>
<tr>
<td>$S_3 = 3a_1 + 3d = 3/2[a_1 + a_3]$</td>
</tr>
<tr>
<td>$S_4 = a_1 + (a_1 + d) + (a_1 + 2d) + (a_1 + 3d) = 4a_1 + 6d$</td>
</tr>
<tr>
<td>$S_4 = 4a_1 + 6d = 4/2[a_1 + a_4]$</td>
</tr>
<tr>
<td>$S_n =$</td>
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</tbody>
</table>
2. Write the formula for the sum of first $n$ terms of an A.P.

\[
S_3 = a_1 + (a_1 + d) + (a_1 + 2d) = 3a_1 + 3d
\]

\[
= 3/2[2a_1 + (3-1)d]
\]

\[
S_4 = 4a_1 + 6d
\]

\[
= 4/2[2a_1 + (4 - 1)d]
\]

\[
S_5 = 5a_1 + 10d
\]

\[
= 5/2[2a_1 + (5 - 1)d]
\]

\[
S_n = \]

3. Observe the given pattern and answer the questions.

\[
1 = 1 = \frac{1 \times (1 + 1)}{2}
\]

\[
1 + 2 = 3 = \frac{2 \times (2 + 1)}{2}
\]

\[
1 + 2 + 3 = 6 = \frac{3 \times (3 + 1)}{2}
\]

\[
1 + 2 + 3 + 4 = 10 = \frac{4 \times (4 + 1)}{2}
\]

What will be the sum of first $n$ terms?

What will be the sum of first 100 terms?

4. Find the sum of first 10 terms of an A.P. whose $n$th term is $2n+3$. What strategy did you apply?
5. Visit the link http://betterexplained.com/articles/techniques-for-adding-the-numbers-1-to-100/ and write your ideas on techniques of adding numbers from 1 to 100.

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## Self Assessment Rubric 14

Content Worksheet (CW9)

**Parameters of assessment**

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<tr>
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<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
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<tbody>
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<td><img src="image3" alt="Box" /></td>
<td><img src="image4" alt="Box" /></td>
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</tbody>
</table>

Able to derive formula for the sum of first $n$ terms of an AP by observing pattern.
Student’s Worksheet 15
Content Worksheet (CW10)
Hands on Activity 2 (Sum of n odd natural numbers)

Name of Student ___________________     Date___________

**Aim:** To verify that "the sum of first n odd natural numbers is $n^2$

**Material Required:** Squared paper, pair of scissors, coloured markers

**Procedure:**
1. Let $n = 10$. Cut a square of dimension 10x10 units from a squared paper as shown below.

![Squared paper with 10x10 grid]

2. To represent 1, shade top left corner square as shown below.

![Shaded square on squared paper]
3. To represent 1 + 3, shade 3 more squares adjacent to previous shaded square as shown below (marked with Red arrows)

4. To represent 1 + 3 + 5, shade 3 more squares adjacent to previous shaded square as shown below (marked with Blue arrows)

5. Repeat shading squares till you represent 1 + 3 + 5 + 7 + 9 + ... + 19 (i.e. sum of first 10 odd natural numbers).

6. What do you observe?

**Answer the following questions:**

1. What is the sum of first 100 odd natural numbers?
2. What is the sum of first p odd natural numbers?
3. Write the sum of odd natural numbers between 5 and 19.
4. Make a geometric representation for 1 + 3 + 5 + 7.
Self Assessment Rubric 15
Content Worksheet  (CW10)

<table>
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<tr>
<th>Parameters of assessment</th>
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</thead>
<tbody>
<tr>
<td>Able to create the visual representation of the sum of first n odd natural numbers.</td>
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<tr>
<td>Able to perform hands on.</td>
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</tbody>
</table>
Do the following

1. The 1st term of an A.P. is 2 and the common difference is 4. Find the sum of (a) First 40 terms (b) First 100 terms.

   First term = ……..
   Common difference = ……………
   Formula for Sum of first n terms = …………

2. The first, second and the last term of an A.P. are t, m and 2t respectively. Find the sum of this sequence.

   Formula for finding sum of first n terms of an A.P. when first term and last term is given as $S_n = ………….$
3. Find the sum of first \( n \) odd natural numbers.

First \( n \) odd natural numbers are: 
First term = 
Common difference = 

4. Find the sum of 20 terms of the A.P. -5, 0, 5, 10 ...

First term = 
Common difference = 
Formula used = 

5. In an A.P. if the 5\(^{th} \) and 12\(^{th} \) terms are 30 and 65 respectively, find the sum of first 25 terms.

\( a_5 = \) 
\( a_{12} = \) 
How will you find \( a \) and \( d \)?
6. Find the sum of all even numbers between 0 and 1000.

Even numbers between 0 and 1000………………
How many are these? …………..

7. Find the sum of all natural numbers between 300 and 700, which are exactly divisible by 7.

Natural numbers between 300 and 700 which are exactly divisible by 7 are : .........................
How many are they?
First term =.........
Last term =.........
Formula for finding sum is $S_n =$

8. Find the sum of first 40 terms of an A.P., whose second term is 2 and seventh term is 22.
9. Find the sum of all odd integers from 3 to 450, which are divisible by 3.

Odd integers from 3 to 450 which are divisible by 3............

10. The sum of 3rd and 7th terms of an A.P. is 6 and their product is 8. Find the sum of first 16 terms.
Self Assessment Rubric 16
Content Worksheet (CW11)

<table>
<thead>
<tr>
<th>Parameters of assessment</th>
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</thead>
<tbody>
<tr>
<td>Able to find the sum of first n terms of an A.P. when first term and common difference is given.</td>
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</tr>
<tr>
<td>Able to find the sum of first n terms of an A.P. when first term, second term and last term is given.</td>
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<tr>
<td>Able to find the sum of first n terms of an A.P. when any two of its terms are given.</td>
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<tr>
<td>Able to find the sum of first n terms of an A.P. under given condition.</td>
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</table>
Post Content Worksheets

PCW1 (Assignment):

1. Is 441 a term of the A.P. : 4, 8, 12, ......? Give reason.
2. Find the 10th term of the A.P. : -40, -30, -20, -10, .......
3. Find the rth term of the A.P. : 1, 3, 5, 7, ....
4. How many terms are there in the A.P. : 8, 14, 20, ...., 206?
5. The first term of an A.P. is 10, the common difference is 10 and the last term is 990. Find the number of terms.
6. The 9th term of an A.P. is 0. Establish a relation between 29th term and 19th term.
7. The 10th term and the 18th term of an A.P. are 41 and 73 respectively. Find the following:
   a) 100th term
   b) Difference between 100th term and 50th term
   c) 7 more than 56th term
   d) 8 less than 6th term
   e) Ratio of 20th term and 25th term
8. Find the 9th term from the end of the A.P. 7, 10, 13, ......187.
9. Two A.P.’s have the same common difference. The difference between their 101th terms is 101. What is the difference between their 201th terms? Give reason for your answer.
10. How many numbers of two digits are divisible by 7?
PCW2 (Brainstorming)

Read the following statements and write your comments.

1. $5n^2 - 6n$ can be the $n$th term of an A.P.

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2. The first term of an A.P. is 7 and the common difference is 5. Meera said 115 is a term of this A.P. What do you say?

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3. The sequence 13, 13, 13, 13, 13... is a G.P.

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4. The common difference of a constant A.P is always zero.

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5. General term of an A.P. is always linear in n.

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6. The sum of consecutive three terms of an A.P. is always double the middle term.

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7. $\sqrt{18}, 6, \sqrt{72}$ is an A.P.

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8. Common difference of two A.P’s can be same or different.

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PCW3 Project- Designs using A.P.


Watch the designs and make designs of your own.

Make a project report also.
PCW4 Framing questions

Using the given picture, frame a question based on the concept of A.P. or G.P.

<table>
<thead>
<tr>
<th>Picture</th>
<th>Question</th>
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<tbody>
<tr>
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<td><img src="image2.png" alt="Picture" /></td>
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<td><img src="image3.png" alt="Picture" /></td>
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<tr>
<td><img src="image4.png" alt="Picture" /></td>
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<td><img src="image5.png" alt="Picture" /></td>
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<td><img src="image6.png" alt="Picture" /></td>
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</tbody>
</table>
**PCW5 Hands on activity**

Aim: To verify that the sum of first \( n \) natural numbers is \( n(n + 1)/2 \), i.e. \( \sum n = n(n + 1)/2 \), by graphical method.

Material Required: Coloured paper, squared paper, and sketch pen, ruler

**Procedure:** Let us consider the sum of natural numbers say from 1 to 10, i.e. \( 1 + 2 + 3 + \ldots + 9 + 10 \). Here \( n = 10 \) and \( n + 1 = 11 \).

1. Take a squared paper of size \( 10 \times 11 \) squares and paste it on a chart paper.
2. On the left side vertical line, mark the squares by 1, 2, 3, \ldots 10 and on the horizontal line, mark the squares by 1, 2, 3 \ldots 11.
3. With the help of sketch pen, shade rectangles of length equal to 1 cm, 2 cm, \ldots, 10 cm and of 1 cm width each.

![Shaded rectangles](image)

Observations: The shaded area is one half of the whole area of the squared paper taken. To see this, cut the shaded portion and place it on the remaining part of the grid. It is observed that it completely covers the grid. Area of the whole squared paper is \( 10 \times 11 \text{ cm}^2 \)

Area of the shaded portion is \( (10 \times 11)/2 \text{ cm}^2 \)

This verifies that, for \( n = 10 \), \( \sum n = n \times (n + 1)/2 \) The same verification can be done for any other value of \( n \).
Answer the following after doing this activity:

1. What is the sum of first 50 natural numbers?
2. What is the sum of natural numbers between 60 and 80?
3. What is the sum of first 100 multiples of 4?
4. What is the sum of first 60 multiples of 9?
## Suggested videos and Extra readings

<table>
<thead>
<tr>
<th>Topic</th>
<th>Link</th>
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<tbody>
<tr>
<td>Number pattern and number sequence</td>
<td><a href="http://mykhmsmathclass.blogspot.com/2011/09/arithmetic-progression-introduction.html">http://mykhmsmathclass.blogspot.com/2011/09/arithmetic-progression-introduction.html</a></td>
</tr>
<tr>
<td>Understanding A.P</td>
<td><a href="http://mykhmsmathclass.blogspot.com/2008/05/arithmetic-progression.html">http://mykhmsmathclass.blogspot.com/2008/05/arithmetic-progression.html</a></td>
</tr>
<tr>
<td>Understanding GP</td>
<td><a href="http://mykhmsmathclass.blogspot.com/search/label/Resources-%20Geometric%20Progression">http://mykhmsmathclass.blogspot.com/search/label/Resources-%20Geometric%20Progression</a></td>
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