PAIR OF LINEAR EQUATIONS IN TWO VARIABLES (CORE)
The CBSE-International is grateful for permission to reproduce and/or translate copyright material used in this publication. The acknowledgements have been included wherever appropriate and sources from where the material has been taken duly mentioned. In case anything has been missed out, the Board will be pleased to rectify the error at the earliest possible opportunity.

All Rights of these documents are reserved. No part of this publication may be reproduced, printed or transmitted in any form without the prior permission of the CBSE-i. This material is meant for the use of schools who are a part of the CBSE-International only.
The Curriculum initiated by Central Board of Secondary Education -International (CBSE-i) is a progressive step in making the educational content and methodology more sensitive and responsive to the global needs. It signifies the emergence of a fresh thought process in imparting a curriculum which would restore the independence of the learner to pursue the learning process in harmony with the existing personal, social and cultural ethos.

The Central Board of Secondary Education has been providing support to the academic needs of the learners worldwide. It has about 11500 schools affiliated to it and over 158 schools situated in more than 23 countries. The Board has always been conscious of the varying needs of the learners in countries abroad and has been working towards contextualizing certain elements of the learning process to the physical, geographical, social and cultural environment in which they are engaged. The International Curriculum being designed by CBSE-i, has been visualized and developed with these requirements in view.

The nucleus of the entire process of constructing the curricular structure is the learner. The objective of the curriculum is to nurture the independence of the learner, given the fact that every learner is unique. The learner has to understand, appreciate, protect and build on values, beliefs and traditional wisdom, make the necessary modifications, improvisations and additions wherever and whenever necessary.

The recent scientific and technological advances have thrown open the gateways of knowledge at an astonishing pace. The speed and methods of assimilating knowledge have put forth many challenges to the educators, forcing them to rethink their approaches for knowledge processing by their learners. In this context, it has become imperative for them to incorporate those skills which will enable the young learners to become 'life long learners'. The ability to stay current, to upgrade skills with emerging technologies, to understand the nuances involved in change management and the relevant life skills have to be a part of the learning domains of the global learners. The CBSE-i curriculum has taken cognizance of these requirements.

The CBSE-i aims to carry forward the basic strength of the Indian system of education while promoting critical and creative thinking skills, effective communication skills, interpersonal and collaborative skills along with information and media skills. There is an inbuilt flexibility in the curriculum, as it provides a foundation and an extension curriculum, in all subject areas to cater to the different pace of learners.

The CBSE has introduced the CBSE-i curriculum in schools affiliated to CBSE at the international level in 2010 and is now introducing it to other affiliated schools who meet the requirements for introducing this curriculum. The focus of CBSE-i is to ensure that the learner is stress-free and committed to active learning. The learner would be evaluated on a continuous and comprehensive basis consequent to the mutual interactions between the teacher and the learner. There are some non-evaluative components in the curriculum which would be commented upon by the teachers and the school. The objective of this part or the core of the curriculum is to scaffold the learning experiences and to relate tacit knowledge with formal knowledge. This would involve trans-disciplinary linkages that would form the core of the learning process. Perspectives, SEWA (Social Empowerment through Work and Action), Life Skills and Research would be the constituents of this 'Core'. The Core skills are the most significant aspects of a learner’s holistic growth and learning curve.

The International Curriculum has been designed keeping in view the foundations of the National Curricular Framework (NCF 2005) NCERT and the experience gathered by the Board over the last seven decades in imparting effective learning to millions of learners, many of whom are now global citizens.

The Board does not interpret this development as an alternative to other curricula existing at the international level, but as an exercise in providing the much needed Indian leadership for global education at the school level. The International Curriculum would evolve on its own, building on learning experiences inside the classroom over a period of time. The Board while addressing the issues of empowerment with the help of the schools’ administering this system strongly recommends that practicing teachers become skillful learners on their own and also transfer their learning experiences to their peers through the interactive platforms provided by the Board.

I profusely thank Shri G. Balasubramanian, former Director (Academics), CBSE, Ms. Abha Adams and her team and Dr. Sadhana Parashar, Head (Innovations and Research) CBSE along with other Education Officers involved in the development and implementation of this material.

The CBSE-i website has already started enabling all stakeholders to participate in this initiative through the discussion forums provided on the portal. Any further suggestions are welcome.

Vineet Joshi
Chairman
Acknowledgements

Advisory
Shri Vineet Joshi, Chairman, CBSE
Shri Shashi Bhushan, Director (Academic), CBSE

Conceptual Framework
Shri G. Balasubramanian, Former Director (Acad), CBSE
Ms. Abha Adams, Consultant, Step-by-Step School, Noida
Dr. Sadhana Parashar, Head (I & R), CBSE

Ideators
Ms. Aditi Misra
Ms. Amita Mishra
Ms. Anita Sharma
Ms. Anita Makkar
Dr. Anju Srivastava
Ms. Anuradha Sen
Ms. Archana Sagar
Ms. Geeta Varshney
Ms. Guneet Ohri
Dr. Indu Khetrapal

English :
Ms. Sarita Manuja
Ms. Renu Anand
Ms. Gayatri Khanna
Ms. P. Rajeshwary
Ms. Neha Sharma
Ms. Sarabjit Kaur
Ms. Ruchika Sachdev

Geography:
Ms. Deepa Kapoor
Ms. Bharti Dave
Ms. Bhagirathi
Ms. Archana Sagar
Ms. Manjari Rattan

Mathematics :
Dr. K.P. Chinda
Mr. J.C. Nijhawan
Ms. Rashmi Kathuria
Ms. Divya Chetal
Ms. Deepa Gupta

Political Science:
Ms. Sharmila Bakshi
Ms. Srelekh Mukherjee

Material Production Groups: Classes IX-X

Science :
Ms. Charu Maini
Ms. S. Anjum
Ms. Meenambika Menon
Ms. Novita Chopra
Ms. Neeta Rastogi
Ms. Pooja Sareen

Economics:
Ms. Mridula Pant
Mr. Pankaj Bhanwani
Ms. Ambica Gulati

Material Production Groups: Classes VI-VIII

History :
Ms. Jayshree Srivastava
Ms. M. Bose
Ms. A. Venkatachalam
Ms. Smita Bhattacharya

Material Production Group: Classes I-V

Dr. Indu Khetarpal
Ms. Vandana Kumar
Ms. Anju Chauhan
Ms. Deepti Verma
Ms. Ritu Batra
Ms. Rupa Chakravarty
Ms. Anuradha Mathur
Ms. Savinder Kaur Rooprai
Ms. Seema Choudhary
Ms. Kalyani Voleti
Ms. Anita Makkar
Ms. Kalpana Mattoo
Ms. Monika Thakur
Mr. Bijo Thomas

Mathematics :
Ms. Seema Rawat
Ms. N. Vidyav
Ms. Mamtta Goyal
Ms. Chhavi Raheja

Political Science:
Ms. Kanu Chopra
Ms. Shilpi Anand

Geography:
Ms. Suparna Sharma
Ms. Leela Grewal
Ms. Leeza Dutta
Ms. Kalpana Pant

Coordinators:
Dr. Sadhana Parashar, Head (I and R)
Dr. Srijata Das, E O (Com)
Dr. Rashmi Sethi, E O (Science)
Shri R. P. Sharma, Consultant
Ms. Ritu Naran, RO (Innovation)
Ms. Sugandh Sharma, E O (Com)
Ms. Sindhu Saxena, R O (Tech)
Ms. Preeti Hans, Proof Reader
Ms. Nandita Mathur
Ms. Seema Chowdhary
Ms. Ruba Chakravarty
Ms. Mahua Bhattacharya

Material Production Group: Classes I-V
1. Syllabus

2. Scope document

3. Teachers' Support Material
   - Teacher's Note
   - Activity - Skill Matrix
   - Warm up Activities
     - W1: Activity 1
     - W2: Activity 1
     - W3: Activity 1
   - Pre content Activities
     - P1: Activity 1
     - P2: Activity 1
   - Graphical Representation of Pair of Linear Equations in Two Variables
     - Activity 1
   - Solution of Pair of Linear Equations in Two Variables Graphically
     - Activity 2
   - Relation between Coeff of x and y
     - Activity 1
   - Condition for Consistent and Inconsistent Systems
     - Activity 1
   - Nature of System of Linear Equations
     - Activity 1
     - Activity 2
     - Activity 3
     - Activity 4
   - Solution by Substitution Method
     - Activity 1
   - Solution by Elimination Method
     - Activity 1
   - Word Search
     - Activity 1
   - Word Problems
     - Activity 1
4. Assessment Guidance Plan
5. Study Materials
6. Students' Support Material

Worksheets and Self-Assessment Rubrics for

- Warm-up W1: Locating a point
- Warm-up W2: Separating the Linear Equation in One Variable and in two Variable
- Warm-up W3: Plotting a point
- Pre-content P1: Basic terms in Linear Equations
- Pre-content P2: Solutions of Linear Equations
- Content CW1: Graphical representation of pair of Linear Equation in two variables.
- Content CW2: Solution of pair of Linear Equations in two variables graphically
- Content CW3: Relation between coff. of x and y and the number of solution for a pair of Linear Equations
- Content CW4: Conditions for Consistent and Inconsistent systems.
- Content CW5: Nature of System of Linear Equations
- Content CW6: Solution by Substitution Method
- Content CW7: Solution by Elimination Method
- Content CW8: Word Search
- Content CW9: Word Problems
• Content CW10: Solution by Cross-multiplication Method
• Post - Content PCW1: Dialogue Dilemma
• Post - Content PCW2: Predicting the number of solutions of equations
• Post - Content PCW3: Cartoon based assignment on Linear Equations
• Post - Content PCW4: Assignments based on different method of solving the pair of Linear Equations
• Post - Content PCW5: Assignment based on Word Problems
• Post - Content PCW6: ICT based projects
• Suggested Videos and Extra Readings
### Syllabus Unit 3:
#### Pair of Linear Equations in two variables (Core)

| Graphical representation of linear Equations | Plotting the lines representing two linear equations on the same plane

Algebraic interpretation of graphs of simultaneous equations as following:

- a) Intersecting lines with common point means linear equation with unique solution
- b) parallel lines with no common point means linear equations with no solution
- c) coinciding lines with all point common means linear equations with infinite solutions

Define: consistent system & inconsistent system |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature of system of linear equations</td>
<td>Relation between the coefficients of pair of linear equations to predict about the given system of linear equations</td>
</tr>
<tr>
<td>Algebraic method of solving system of linear equations</td>
<td>Substitution method, elimination method and cross multiplication method.</td>
</tr>
<tr>
<td>Application in daily life problems</td>
<td>Number problems, age problems, work ratio problems, dimensional problems.</td>
</tr>
</tbody>
</table>
Key terms

1. Pair of linear equations
2. Graphical representation of pair of linear equations
3. Algebraic interpretation of Graphical representation of pair of linear equations
4. Nature of system of linear equations
5. Consistent system of pair of linear equations
6. Inconsistent system of pair of linear equations
7. Substitution method of solving pair of linear equations
8. Cross-multiplication method of solving pair of linear equations
9. Elimination method of solving pair of linear equations

Learning Objective:

- Plot the lines representing the linear equations of given system on same plane.
- Observe that intersecting lines have one common point, coinciding lines have all points common and parallel lines have no common point
- Understand the Algebraic interpretation of graphical representation i.e. intersecting lines implies unique solution, coinciding lines implies infinite solution and parallel lines implies no solution
- Observe the coefficients of system of linear equations $a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0$ and their graphs and establish the following relation:
  1. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ for unique solution
  2. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ for infinite solution
  3. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ for no solution
- Grasp the terms consistent system and inconsistent system of linear equations
- Predict the nature of the system of linear equations looking at the coefficients
- Solving the pair of linear equations by substitution method
- Solving the pair of linear equations by elimination method
- Solving the pair of linear equations by cross-multiplication method
- Solving word problems based on the real life situation

**Extension activities:**

- Use Geo-Gebra and explore what kind of lines are obtained for the pair of linear equations \( ax + by + c = 0 \), \( dx + ey + f = 0 \)
  - when \( \frac{a}{d} \neq \frac{b}{e} \neq \frac{c}{f} \)
  - when \( \frac{a}{d} \neq \frac{b}{e} \), but \( \frac{a}{d} = \frac{c}{f} \)
  - when \( \frac{a}{d} \neq \frac{b}{e} \), but \( \frac{b}{e} = \frac{c}{f} \)

**Life skill:**

Check the meaning of the word consistent and inconsistent from the dictionary and find out success stories of consistent performers. Discuss the importance of being consistent performer in life.

**Perspective:**

What kind of graph will be obtained for linear equation in three variables? How the solution will look graphically? Get some hint from following picture taken from Wikipedia.
Every day various schemes or plans are launched by mobile companies. Use your knowledge of framing and solving pair of linear equations and compare the plans. Educate your friends and parents about making choices before availing any plan. This knowledge can be applied to compare all kind of schemes whether it is regarding insurance companies or taking home loan or purchasing car in instalments.
Teachers’ Support Material
TEACHER’S NOTE

The teaching of Mathematics should enhance the child’s resources to think and reason, to visualise and handle abstractions, to formulate and solve problems. As per NCF 2005, the vision for school Mathematics include:

1. Children learn to enjoy mathematics rather than fear it.
2. Children see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.
3. Children pose and solve meaningful problems.
4. Children use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth or falsity of statements.
5. Children understand the basic structure of Mathematics: Arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathematics, all offer a methodology for abstraction, structuration and generalisation.
6. Teachers engage every child in class with the conviction that everyone can learn mathematics.

Students should be encouraged to solve problems through different methods like abstraction, quantification, analogy, case analysis, reduction to simpler situations, even guess-and-verify exercises during different stages of school. This will enrich the students and help them to understand that a problem can be approached by a variety of methods for solving it. School mathematics should also play an important role in developing the useful skill of estimation of quantities and approximating solutions. Development of visualisation and representations skills should be integral to Mathematics teaching. There is also a need to make connections between Mathematics and other subjects of study. When children learn to draw a graph, they should be encouraged to perceive the importance of graph in the teaching of Science, Social Science and other areas of study. Mathematics should help in developing the reasoning skills of students. Proof is a process which encourages systematic way of argumentation. The aim should be to develop arguments, to evaluate arguments, to make conjunctures and understand that there are various methods of reasoning. Students should be made to understand that mathematical communication is precise, employs unambiguous use of language and rigour in formulation. Children should be encouraged to appreciate its significance.

At the secondary stage students begin to perceive the structure of Mathematics as a discipline. By this stage they should become familiar with the characteristics of Mathematical communications, various terms and concepts, the use of symbols, precision of language and systematic arguments in proving the proposition. At this stage a student should be able to integrate the many concepts and skills that he/she has learnt in solving problems.
The unit on “Linear Equations in Two Variables” focuses on lots of geogebraic activities and exploration in order to meet out the following learning objectives:

- Plot the lines representing the linear equations of given system on same plane.
- Observe that intersecting lines have one common point, coinciding lines have all points common and parallel lines have no common point.
- Understand the Algebraic interpretation of graphical representation i. e. intersecting lines implies unique solution, coinciding lines implies infinite solution and parallel lines implies no solution.
- Observe the coefficients of system of linear equations \( a_1x+b_1y+c_1=0 \) & \( a_2x+b_2y+c_2=0 \) and their graphs and establish the following relation:
  4. \( \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \) for unique solution
  5. \( \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \) for infinite solution
  6. \( \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \) for no solution
- Grasp the terms consistent system and inconsistent system of linear equations.
- Predict the nature of the system of linear equations looking at the coefficients.
- Solving a pair of linear equations by substitution method.
- Solving a pair of linear equations by elimination method.
- Solving a pair of linear equations by cross-multiplication method.
- Solving word problems based on real life situation.

All the tasks designed in this chapter have been prepared keeping in mind the following pedagogical issues:

- To create supportive classroom environment in which learners can think together, learn together, participate in the discussions and can take intellectual decisions.
- To provide enough opportunities to each learner of “expression” so that teacher can have insight into the knowledge acquired, knowledge required,
refinement required in the knowledge gained and the thinking process of the learner.

- Emphasis on creating a good communicative environment in the class.
- To cater various learning styles.

Pair of linear equations in two variables is a unit in which both algebra and coordinate geometry shares equal role. Using algebra, a real life problem can be translated into abstract expression involving coefficients and variables. The algebraic problems can be solved either using coordinate geometry (graph of linear equations) or through pure algebraic approach.

The study of this unit requires a clear understanding of basic concepts of coordinate geometry i.e. ordered pair, Cartesian plane, Cartesian coordinates, abscissa, coordinate axis, skill of drawing lines in Cartesian plane, etc. and the basic concepts of algebra like algebraic expressions, variables, coefficients, linear equations in one variable, solution of linear equation, linear equations in two variables, etc.

To attain the first learning objective various warm up activities can be used rather than giving instructions to directly plot the lines. In the warm up activity (W 1) some paper slips with ordered pair written over it will be kept on hold. Students can pick up the slips and speak anything which comes to their mind related to the location of given ordered pair. The idea behind this exercise is to involve all the learners. No prescribed format is given to describe the ordered pair. Learner can give any appropriate statement. Teacher can also ask the students to simultaneously point out the quadrant on the Cartesian plane drawn on blackboard/screen. This exercise will help all the learners to brush up their knowledge of locating the point on plane. If any learner is lacking in the concept then with the repeat of same exercise, every child will be able to locate the point in the plane. After this exercise a self assessment sheet with four parameters will be distributed to every learner. With the help of this self assessment rubric they can make out whether they can identify the points located on X-axis/Y-axis/origin/any of the four quadrants. In the same way warm up exercise (W3) will help the students to plot the given ordered pairs on the same plane followed by
the self assessment rubric. Warm up activity (W2) aims at brushing up the concept of algebraic expressions, linear equations, coefficients and variables. During this activity teacher will encourage the students to speak about the written algebraic statement on the board/screen. Teacher can motivate the students to frame some real life situations where the written equation or expression can make sense. This way the students will be able to relate abstract mathematics with day to day acquaintance and will be motivated to learn more about this unit. Moreover, for the same algebraic expression, different situations can be quoted. This will help the group to widen their horizon and their perspective to assimilate the ideas. Some students always hesitate to share their thoughts due to lack of confidence or due to the feeling that one right answer been given and no other right answer is possible. The above situation can be used to remove the blockage from the mind of the students and to help them understand that there could be several right answers or various possible solutions to the same problem. This way, not only the thinking skills of the students would be developed, learners will also tend to do divergent thinking. Pre-content tasks (P1) and (P2) also covers up the pre-requisites for plotting the lines on Cartesian plane. Before taking up the Content worksheet (CW 1) in the class, teacher can conduct a discussion in the class regarding the number of points lying on a line and the number of solutions which are possible for a linear equation. Brain storming questions can be asked in the class after the students have plotted a line for a given linear equation.

Let the students observe the graph and answer the following questions.

1. How many points can be located on a line?
2. Identify any one solution of the given linear equation.
3. How many more solutions you can find out?
4. Is there any correspondence between the points located on the line and the solutions of the linear equations?

Students can gradually explore, what will happen if two linear equations are plotted on the same Cartesian plane. Teacher can give different pair of linear equations and ask the students to plot the lines on the same plane. Further they
can be motivated to observe the various possible relationships between the lines. Through discussion it should come out from students that there are only three possibilities between two given lines:

1. Two lines can be intersecting, or
2. Two lines can be parallel, or
3. Two lines can be coinciding.

Discussion can be conducted to make students think the possible values of variables which can satisfy both the given equations simultaneously. Let the students explore the relationship between these possible values/solution of the equations and the lines representing them on the Cartesian plane. Once the students have understood that intersecting lines means unique solution, coinciding lines means infinite solutions and parallel lines means no-solution, new terms consistent system and inconsistent system can be introduced. Apart from the vocabulary development, the words can be used to inculcate life skills attitudes regarding work culture amongst the students. A healthy debate can be conducted in the class regarding the advantages of being a consistent worker or disadvantages of being an inconsistent worker. A discussion can also be held to sensitize the students how the CCE system introduced in the schools is preparing the child to be a consistent performer. Consistent performance is an essential requisite of modern day industry. So the students need to develop in themselves the attitude of working consistently and to avoid fluctuations in mood at work place.

After attempting a number of problems and using geogebra students can be triggered to think beyond and can be motivated to find the area of a triangle formed by an axis and a pair of lines. They can also try to

1. Find out how many triangles are possible by two lines and an axis?
2. What are the coordinates of the points when the two lines meet at X-axis or Y-axis?
A brainstorming session can be conducted to improve the skills of finding the solution of a pair of linear equations in two variables when some values of one variable are unknown corresponding to the known values of the other variables.

To strengthen the concepts and to help the learners to learn at their own pace a lot of videos available on you-tube can be suggested as mentioned in teachers support material (activity 6 – content (CW1)). Extra reading material is also suggested to establish them as independent learners.

Further, students should be motivated to see the relationship between the coefficients of these equations. Through numerous examples allow them to investigate the relationship between the ratios of coefficients of two variables x and y as well as constant terms. Students should be able to reach the conclusion that for a given pair of equations $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$

**Conditions:**

1. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ for unique solution
2. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ for infinite solution
3. $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ for no solution

Using geogebra activities the learning can be extended to explore the relation between the lines when

1. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
2. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ but } \frac{a_1}{a_2} = \frac{c_1}{c_2}$
3. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ but } \frac{b_1}{b_2} = \frac{c_1}{c_2}$

By observing the ratio of the coefficients students can make out that when system of equations has infinite solutions the equations are dependent on each other. So, new terms like independent system and dependent system can be introduced. With all the above exercises, although self assessment rubric for the students are suggested at the end of each worksheet, to reinforce and to
strengthen the concepts, content work sheet (CW 4) and (CW 5) focus on assimilation of all these concepts.

Beauty of mathematics lies in the fact that there are various paths to reach the same conclusion. Same is true for finding the solution of linear equations. Linear equations can be solved algebraically too without using any graph. Various methods like substitution, elimination, cross multiplication method can be used to do so. Each method can be explained to the students with the help of examples and can be mastered by them by attempting a good number of problems. Students can also be motivated to solve the general equations \( a_1 x + b_1 y + c_1 = 0 \) and \( a_2 x + b_2 y + c_2 = 0 \) and to generalize the solutions as formulae.

To revise all the concepts and to strengthen the vocabulary related to the chapter, word search in the form of content worksheet (CW 8) is given. This activity is framed in order to break the monotony of drilling the problem solve. An interesting activity, dialogue dilemma with, the help of cartoon strips, can be conducted in the class. After reading the dialogues between various characters illustrated in cartoon strips students will give answers based on all concepts learnt in the chapter.

To achieve the last learning objective i.e. to solve real life problems with the help of linear equations, students can be asked to pose some situations which can be translated into linear equations. They can be given certain situations and asked to convert them into linear equations and gradually can be motivated to solve them using any of the above learnt methods. Throughout the development of the chapter teacher should keep in mind not to introduce any term or concept directly. Focus should be on taking the students on a journey of exploration where they can use their previous knowledge and identify the new relations. Although geogebra software is recommended for all explorations but in case of unavailability it should not be taken as drawback. Rather student should be encouraged to draw the graphs, sit in groups and explore the desired results. Teachers are also suggested not to discourage any child for asking unusual question or giving unusual response. Some students can ask what will happen if the linear equations contains more than two variables. They can be encouraged
to find more about linear equations in three or more variables with the help of geogebra or the information available on web. Teacher should divert the response in right direction by asking appropriate questions.

At the end students should also be encouraged to identify the areas where they can use the knowledge acquired in this chapter. For example, the advertisements given by various agencies in newspapers and magazines to avail the car loan or the house loan etc. can be compared using knowledge of solving a pair of linear equations. Overall the entire unit can be used to enhance exploratory skills, to enhance divergent kind of thinking, to find the relation between the abstract and practical situations and to nurture creativity along with inculcating the values regarding work culture required for 21st century.
<table>
<thead>
<tr>
<th>Type of Activity</th>
<th>Name of Activity</th>
<th>Skill to be developed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Warm UP (W1)</td>
<td>Locating a point</td>
<td>Spatial skill, geometrical skill, drawing of graph</td>
</tr>
<tr>
<td>Warm UP (W2)</td>
<td>Separating the linear equations in one variable and in two variables</td>
<td>Knowledge and understanding</td>
</tr>
<tr>
<td>Warm UP (W3)</td>
<td>Plotting a point</td>
<td>Geometrical skill, drawing of graph</td>
</tr>
<tr>
<td>Pre-Content (P1)</td>
<td>Basic terms in linear equation</td>
<td>Knowledge and understanding</td>
</tr>
<tr>
<td>Pre-Content (P2)</td>
<td>Solution of linear equation</td>
<td>Synthesis of information</td>
</tr>
<tr>
<td>Content (CW 1)</td>
<td>Graphical representation of pair of linear equation in two variables</td>
<td>Spatial skill, geometrical skill, drawing of graph</td>
</tr>
<tr>
<td>Content (CW 2)</td>
<td>Solution of pair of linear equations in two variables graphically</td>
<td>Drawing and interpretation of graph</td>
</tr>
<tr>
<td>Content (CW 3)</td>
<td>Relation between coefficient of x and y and number of solutions for a pair of linear equation</td>
<td>Application and computational skills</td>
</tr>
<tr>
<td>Content (CW 4)</td>
<td>Conditions for consistent and inconsistent systems</td>
<td>Memory, knowledge, understanding an application</td>
</tr>
<tr>
<td>Content (CW 5)</td>
<td>Nature of</td>
<td>Thinking skills</td>
</tr>
<tr>
<td>Content (CW 6)</td>
<td>Substitution method</td>
<td>Computational skills.</td>
</tr>
<tr>
<td>Content (CW 7)</td>
<td>Elimination Method</td>
<td>Computational skills.</td>
</tr>
<tr>
<td>Content (CW 8)</td>
<td>Word search</td>
<td>Memory and observation</td>
</tr>
<tr>
<td>Content (CW 9)</td>
<td>Word problems</td>
<td>Problem solving skills, Thinking skill and computational skills</td>
</tr>
<tr>
<td>Content (CW 10)</td>
<td>Cross-multiplication method</td>
<td>Computational skills</td>
</tr>
<tr>
<td>Post - Content (PCW 1)</td>
<td>Dialogue Dilemma</td>
<td>Analytical skills, communication skills, thinking skills.</td>
</tr>
<tr>
<td>Post - Content (PCW 2)</td>
<td>Predicting the number of solution of equations</td>
<td>Computational skills.</td>
</tr>
<tr>
<td>Post - Content (PCW 3)</td>
<td>Cartoon based assignment on linear equations</td>
<td>Observation, expression, knowledge and comprehension</td>
</tr>
<tr>
<td>Post - Content (PCW 4)</td>
<td>Assignment based on different methods of solving the pair of linear equations</td>
<td>Problem solving, computational skills</td>
</tr>
<tr>
<td>Post - Content (PCW 5)</td>
<td>Assignment based on word problems</td>
<td>Problem solving, computational skills</td>
</tr>
<tr>
<td>Post - Content (PCW 6)</td>
<td>ICT based project</td>
<td>e-skills, observation skills, Analytical skills and synthetic skill, organizing and compiling of data</td>
</tr>
</tbody>
</table>
Activity 1 – warm up (W1)

Specific objective:
To recall the location of the ordered pairs in the Cartesian plane

Description: In the earlier classes the students gained the knowledge of quadrants in the Cartesian plane. This is a starter activity through which the students will be motivated for learning the plotting of ordered pairs in their respective quadrants. Each student will speak about the respective ordered pair, its location in the quadrant/x-axis/ y- axis/ origin.

Execution: Take some paper slips and write an ordered pair on each. Put all of them in a bowl. Each student will pick up one paper slip and tell the location of ordered pairs in the Cartesian plane.

Note: Lay stress on different locations in the Cartesian plane. The ordered pair (0, 0) is origin. A point on the x- axis is of type (x, 0) and on the y- axis is of the type (0, y). Talk about the signs of abscissa and ordinates in four quadrants.

Parameters for assessment:

Students are able to tell

- Location of a point in four quadrants
- Location of a point on the x- axis
- Location of a point on the y - axis
- Location of the Origin.

Extra reading:

http://www.webmath.com/gpoints.html
Video Watch: You may ask the students to watch video http://www.youtube.com/watch?v=-Hut9QnQIF8&NR=1&feature=fvwp for revision of plotting of points in a Cartesian plane.

Activity 2 – warm up (W2)

Specific objective: To recall linear equation in one variable or two variables.

Description: In earlier classes the students learnt of the difference between an algebraic expression and an equation. Through this warm up activity they will brushup their knowledge by recognizing the algebraic expressions and linear equations.

Execution: Teacher will write some algebraic expressions, linear equations in one variable and two variables. Students will speak out whether it is an algebraic expression or linear equation in one variable or two variables.

Note: To make the task more interesting you may prepare flash cards and use them in the class.

Parameters for assessment:

Students are able to

- Identify a linear equation in one – variable.
- Identify a linear equation in two – variables.
Activity 3 – warm up (W3)

Specific objective: To recall the knowledge of Cartesian plane and locate the ordered pairs shown on the Cartesian plane.

Description: In the earlier classes the students have gained the knowledge of all the four quadrants formed by the two axes.

They will use their knowledge and the concept of quadrants to locate the ordered pairs in their respective quadrant.

Execution: Distribute the worksheets and ask the students to locate the ordered pairs on the graph.

Parameters for assessment:

- Location of a point in a quadrant
- Location of a point on the x-axis
- Location of a point on the y-axis
- Location of Origin.

Activity 4 – Pre content (P1)

Specific objective: To make the students recall the terms coefficients, variables, constant term of a linear equation in two variables

Description: This is a pre content activity for refreshing students knowledge about various terms related to a linear equation in two variables. Using the given coefficients and the given statements students will convert the statements into Mathematical expressions. Students can also be encouraged to narrate some situations corresponding to the obtained linear equation. Teacher must help the students to realize that to such kind of questions more than one answers are possible.

Execution: Teacher may ask the students to write a linear equation according to the given condition. There can be varied answers. Let students speak their answers and discuss them.

Parameters for assessment

Student has the knowledge of

❖ Terms like coefficients, variables, constant term, linear equation

Extra reading:

http://www.mbs.edu/home/jgans/mecon/value/Popups/pop_up_analytical_methods.htm
Activity 5 – Pre content (P2)

Specific objective: To make the students recall what a solution of an equation is?

Description: In class 9 students have learnt to find solutions of a linear equation in one variable as well as linear equation in two variables. They will be motivated predict the solution of a linear equation in two variables. Each student will check if the given ordered pair satisfies the given linear equation in two variables.

Execution: After distributing the worksheets, students will be asked to substitute the abscissa and ordinate of the ordered pair in the given equation and check if it satisfies the equation. If it satisfies then it is a solution otherwise not.

Points to note: There can be more than one solution of an equation. All the points which are solution of the given equation will lie on the graph of the equation. Discussion can be held to make the learners realize that a linear equation in two variables can have infinite solutions. Each solution will correspond to a point on the line representing linear equation in two variables.

Parameters for assessment

- Able to find solution of a linear equation in two variables

Follow up: Watch Video

http://www.youtube.com/watch?v=cHH_NqNuwYI
Activity 6 – Content (CW1)

Specific objective:

• To learn to plot a pair of linear equations in two variables on the graph

Description: In this content activity, the students are encouraged to plot the coordinates (ordered pairs) on the graph, given in each table and to join them and identify what they get. They have to locate more points on it and identify how many more points can be there on it?

Discuss the Key Words: Coincident, Parallel, Intersecting

Points to remember: Stress on the following points

• Label the axes.
• Put arrows on the line ends.
• Ordered pairs are marked with capital letters and written as (x, y). Here ‘x’ is abscissa and ‘y’ is ordinate.

Execution: Each student plots the coordinates of each table and joins them. They see what type of lines do they get, do they have any point common or not.

Parameters for assessment:

❖ Locate the points on the Cartesian plane.
❖ Able to join the points and obtain the lines.
❖ Identify types of lines obtained.
❖ Locate the common points on the two lines.

Follow up: You may ask the students to watch videos

http://www.youtube.com/watch?v=VKqledd8wUA

http://www.youtube.com/watch?v=4h65y9Xj4eY&feature=related
Extra reading:

Graphing linear equations in two variables

http://www.tpub.com/math1/13.htm

http://www.puremath.com/modules/graphlin.htm

Video:

http://www.youtube.com/watch?v=jpVrLZdluW0

http://www.mathexpression.com/graphing.html

Brainstorming questions for discussion:

1. The table shows four pairs of values of x and y that satisfy the linear equation
   \[ y = 2x - 3 \]. Find the values for m and n.

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>-1</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>-3</td>
<td>m</td>
<td>-5</td>
<td>n</td>
</tr>
</tbody>
</table>
2. The graph shows five points A, B, C, D and E. Which of the 3 points when connected together will satisfy the linear equation of \( x - y = 0 \)?

**Activity 7 – Content Worksheet (CW 2)**

**Specific objective**: To learn to represent the linear equations in two variables on the graph and read the solution of the pair of equations on the basis of intersecting lines, coincident lines or parallel lines.

**Description**: In the earlier classes the students have learned to draw the graph of a linear equation in two variables. The concept of solution of linear equations will be built up by taking more pair of lines and draw the graphs. Further they will obtain the solution(s), if any.

**Execution:**

Each student will be given a worksheet to find the ordered pairs which satisfies each equation separately. On the provided graph sheet students will draw the graph by joining the ordered pairs obtained. Then the type of lines will be identified and the common points (if any) are found. On the basis of it the system will be termed consistent or inconsistent.
Key words for discussion: Intersecting, Coincident, Parallel, Consistent and Inconsistent

Parameters:

- Finding suitable ordered pairs
- Locate the points on the Cartesian plane.
- Join the points correctly.
- Identify the types of lines correctly.

Extra reading:

http://www.purplemath.com/modules/systlin1.htm
http://www.purplemath.com/modules/systlin2.htm

Brainstorming questions for discussions:

Given below is a graph representing pair of linear equations in two variables. 
\( x+y=4 \), \( 3x-2y=12 \)
Observe the following carefully...
The given two lines intersect at (4,0) which is the solution of given pair of linear equations in two variables.

Coordinates of points where lines cut the y-axis are A (0,4) and C(0,-6)

Vertices of triangle formed by given lines and y-axis are A(0,4), B(4,0) and C(0,-6)

The area of triangle ABC = $\frac{1}{2} (10 \times 4) = 20$ square units

Given below is the graph representing pair of linear equations in two variables

\[ x-y=4, x-2y=4 \]

The given two lines intersect at (4,0) which is the solution of given pair of linear equations in two variables. Coordinates of points where lines cut the y-axis are A (0,-4) and C(0,-2) Vertices of triangle formed by given lines and y-axis are A(0,-4), B(4,0) and C(0,-2) The area of triangle ABC = $\frac{1}{2} (2 \times 4) = 4$ square units

Given below is a graph representing pair of linear equations in two variables \( x-y=2, x+y=4 \) Answer the following questions:
1. What are the coordinates of points where two lines meet x-axis?
2. What are the coordinates of points where two lines meet y-axis?
3. What is the solution of given pair of equations? Read from graph.
4. What is the area of triangle formed by given lines and x-axis?
5. What is the area of triangle formed by given lines and y-axis?
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(x - y = \frac{1}{16} = \frac{4}{4})</td>
<td>2.</td>
</tr>
<tr>
<td>(x - y = 4)</td>
<td>5x - 4y = 25</td>
<td>10y = 74 - 8x</td>
</tr>
<tr>
<td>4.</td>
<td>(8 = -4)</td>
<td>5.</td>
</tr>
<tr>
<td>(y = \frac{1}{4}x - \frac{13}{4})</td>
<td>-4x = 8 - 2y</td>
<td>(y = \frac{4}{3}x + \frac{2}{3})</td>
</tr>
</tbody>
</table>

Created using [http://www.edhelper.com/LinearEquations.htm](http://www.edhelper.com/LinearEquations.htm)
Activity 8 – Content Worksheet (C W 3)

Specific objective:

- The students will be able to identify the coefficients of the variables and constant terms in the given pair of equations.
- They will be able to find a relation between the ratios of the corresponding coefficients of the two variables and the constant terms.

Description:

The students are already aware of the term coefficient of variable and constant terms, they will further identify the coefficient of the variables in both equations and then find their ratio and find a relation between these ratios when lines have unique solution, no solution and many solutions. They will also investigate the types of the graph and the relation between the ratios of the coefficients and constant terms.

Execution:

Each student will identify the coefficients of the variables and the constant terms and find the ratio of coefficients of corresponding variables in two equations. Students will be asked to observe the given graph of a pair of linear equation and compare the ratios of coefficients of x, ratio of coefficient of y and constant terms. From the graphs they will investigate the conditions for consistency and inconsistency for given system.

Parameters:

- Identify the coefficients of the variables.
- Identify the constant terms.
- Calculate the ratio $\frac{a_1}{a_2} \cdot \frac{b_1}{b_2} \cdot \frac{c_1}{c_2}$
- Identify the type of pair of linear equations.
- Identify the number of solution(s).
Activity 9 – Content Worksheet (CW 4)

**Specific objective:** To learn about consistent and inconsistent pair of equations

**Description:** It is an activity followed by the learning of various types of graphical representations of pair of equations and various types of solutions.

**Execution:** Ask the students to recall the types of graphs obtained viz. intersecting, parallel and coincident lines. There are two cases when there is at least one solution. Let students discuss all the cases and the fact that when we are able to find two solutions for a given pair of equations then we will have infinite solutions. In such a case we will get coincident lines.

After explaining the terms consistent and inconsistent system, students will be given Content worksheet (CW4). Students will solve the questions and afterwards there will be a discussion on the answers.

**Parameters:**
- Explanation of two types of systems viz. consistent and inconsistent
- Writing pair of consistent equations
- Writing pair of inconsistent equations

**Extra reading:**

Follow up: Video Watch [http://www.youtube.com/watch?v=R-4FkXOgDQc](http://www.youtube.com/watch?v=R-4FkXOgDQc)

[http://www.youtube.com/watch?v=VqWfxtc2vCg&feature=related](http://www.youtube.com/watch?v=VqWfxtc2vCg&feature=related)
Activity 10 – Content Worksheet (CW 5)

**Specific objective:** To be able to identify the relationship between the ratio of the coefficients and constant with type of lines.

**Description:** In earlier classes the students have already learnt the relation between the ratio of coefficients and constants with the type of pair of linear equations in two variables. In CW 5 they are further given situations in which they will be asked to form equations which will be consistent or inconsistent to the given equation.

**Execution:** Recall, the previous class knowledge and encourage each student to formulate another equation or completes the equations with constants depending upon the type of lines or type of system (consistent or inconsistent)

**Parameters:**

- Writing consistent pairs of equations
- Writing inconsistent pairs of equations
Activity 11 – Content Worksheet (CW 6)

Specific objective: To learn to solve pair of linear equations algebraically – using substitution method.

Description: In CW 6 the teacher will ask the students to express one variable from any one equation in terms of the other. Many examples will be discussed to grill the concept. Further it will be explained to students that if that expression is substituted in the other equation then it becomes an equation in one variable which can be solved easily.

Execution:

Each student will express one variable in terms of the other form first equation and substitute its value in the other to solve the second equation, which becomes an equation in one variable. The solution of it will give the student the value of the first variable.

Parameters:

- Expressing one variable in terms of other.
- Substituting for expression in the other equation.
- Solving the second equation.
- Getting the value of first variable.
- Finding the solution (if any)

Extra reading:


Follow up: Video Watch

http://www.youtube.com/watch?v=cwHR_B9zK7k&feature=related
Activity 12 – Content Worksheet (CW 7)

Specific objective:

Students will be able to solve pair of linear equations algebraically – using elimination method.

Description:

Students are aware of solving pair of linear equations in two variables by substitution method. Moving ahead teacher will ask students to identify the coefficients of the variables. The teacher will explain that we can eliminate one variable if the coefficients of the variable are same in both equations, by subtracting the equations. In this manner, one variable will be eliminated and value of other can be calculated.

Execution:

Students will make the coefficients of either of the variables same by multiplying the whole equation by a certain constant (so that the result is LCM of the coefficients). Then the students will subtract the equations and eliminate one variable and find the value of the other. By substituting its value in any one equation the student will obtain the eliminated variable’s value.

Parameters:

- Make coefficient of one variable same.
- Subtract/add the equations.
- Obtain the value of each variable.
- Gets the value of first variable.

Extra reading:  http://rachel5nj.tripod.com/NOTC/ssoewog2.html
http://www.purplemath.com/modules/systlin5.htm
Activity 13 – Content Worksheet (CW 8)

Specific objective: To recapitulate the various terms learnt in the chapter.

Description: This is a word search fun activity. It is a time bound activity.

Execution: Each student will be given the puzzle template and asked to search the words learnt in the chapters which are placed horizontally, diagonally, vertically as well as backwards. All the students will be given 15 minutes for the activity.

Parameters:

- Finding the correct words

Solution

You may create a word search using the online tool: [http://www.armoredpenguin.com/wordsearch/]
Activity 14 – Content Worksheet (CW 9)

**Specific objective:** To learn to apply the knowledge of pair of linear equations in solving problems.

**Description:** After learning to solve the pair of linear equations in two variables using different methods, application part will be explored through various types of word problems. Initially students will be asked to frame mathematical expressions from small statements and then frame a pair of equations.

**Execution:** Through discussing various types of word problems, the application of pair of equations will be appreciated.

**Parameters:**
- Able to frame mathematical equations
- Able to solve the pair of equations

**Extra reading:** [http://www.purplemath.com/modules/systprob.htm](http://www.purplemath.com/modules/systprob.htm)
Activity 15 – Content Worksheet (CW 10)

Specific objective: To learn to use the cross multiplication method for solving the given pair of linear equations in two variables

Description: After learning to solve the pair of linear equations in two variables using different methods, viz. substitution methods, elimination method, students will be given an exposure to solve the pair of linear equations using cross multiplication method.

Execution: Explain the method and discuss. Distribute the worksheet to students to explore the method by solving some problems using cross multiplication method.

Parameters:
- Able to solve the problems using cross multiplication method

Post Content Activities

Activity 16 – Post Content Worksheet (PCW1) –
Activity 17 – Post Content Worksheet (PCW2) –
Activity 18 – Post Content Worksheet (PCW3) –
Activity 19 – Post Content Worksheet (PCW4) –
Activity 20 – Post Content Worksheet (PCW5) –
Activity 21 – Post Content Worksheet (PCW6) –
Assessment Plan

Assessment guidance plan for teachers

With each task in student support material a self-assessment rubric is attached for students. Discuss with the students how each rubric can help them to keep in tune their own progress. These rubrics are meant to develop the learner as the self motivated learner.

To assess the students’ progress by teacher two types of rubrics are suggested below, one is for formative assessment and one is for summative assessment.

Suggestive Rubric for Formative Assessment (exemplary)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mastered</th>
<th>Developing</th>
<th>Needs motivation</th>
<th>Needs personal attention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic method of solving pair of linear equations in two variables x and y</td>
<td>Able to apply substitution method to solve the pair of linear equations, get correct value of x and y, can verify the correctness of solution</td>
<td>Able to apply substitution method to solve the pair of linear equations, get correct value of x and y, cannot verify the correctness of solution</td>
<td>Able to apply substitution method to solve the pair of linear equations, get correct value of x and y, cannot verify the correctness of solution</td>
<td>Not Able to apply substitution method to solve the pair of linear equations.</td>
</tr>
<tr>
<td>Able to apply elimination method to solve the pair of linear equations, get correct value of x and y, can verify the</td>
<td>Able to apply elimination method to solve the pair of linear equations, get correct value of x and y, cannot verify the correctness of solution</td>
<td>Able to apply elimination method to solve the pair of linear equations, get correct value of x and y, cannot verify the correctness of solution</td>
<td>Able to apply elimination method to solve the pair of linear equations, get correct value of x and y, cannot verify the correctness of solution</td>
<td>Not Able to apply elimination method to solve the pair of linear equations.</td>
</tr>
<tr>
<td>correctness of solution</td>
<td>verify the correctness of solution</td>
<td>cannot verify the correctness of solution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td>-----------------------------------</td>
<td>------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to apply cross-multiplication method to solve the pair of linear equations, get correct value of x and y, can verify the correctness of solution</td>
<td>Able to apply cross-multiplication method to solve the pair of linear equations, get correct value of x and y, cannot verify the correctness of solution</td>
<td>Able to apply cross-multiplication method to solve the pair of linear equations, can not get correct value of x and y,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not able to apply cross-multiplication method to solve the pair of linear equations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From above rubric it is very clear that

- Learner requiring personal attention is poor in concepts and requires the training of basic concepts before moving further.
- Learner requiring motivation has basic concepts but face problem in calculations or in making decision about suitable substitution etc. He can be provided with remedial worksheets containing solution methods of given problems in the form of fill-ups.
- Learner who is developing is able to choose suitable method of solving the problem and is able to get the required answer too. May have the habit of doing things to the stage where the desired targets can be achieved, but avoid going into finer details or to work further to improve the results. Learner at this stage may not have any mathematical problem but may have attitudinal problem. Mathematics teachers can avail the occasion to bring positive attitudinal changes in students’ personality.
- Learner who has mastered has acquired all types of skills required to solve the pair of linear equations in two variables.
### Teachers’ Rubric for Summative Assessment of the Unit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solving pair of linear equations graphically</td>
<td>• Able to express one variable in terms of other variable</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to express one variable in terms of other variable</td>
</tr>
<tr>
<td></td>
<td>• Able to draw table for different values of x and y</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to draw table for different values of x and y</td>
</tr>
<tr>
<td></td>
<td>• Able to plot all points on coordinate plan accurately and get the lines by joining them</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to plot all points on coordinate plan accurately and get the lines by joining them</td>
</tr>
<tr>
<td></td>
<td>• Able to predict the number of solutions according to lines as unique solution/ no solution/ infinite solution.</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to predict the number of solutions according to lines as unique solution/ no solution/ infinite solution.</td>
</tr>
<tr>
<td></td>
<td>• Able to identify the system as consistent or inconsistent</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to identify the system as consistent or inconsistent.</td>
</tr>
<tr>
<td></td>
<td>• Able to read the exact point of intersection and find the solution.</td>
<td></td>
<td></td>
<td></td>
<td>• Not able to read the exact point of intersection and find the solution.</td>
</tr>
</tbody>
</table>
| Solving pair of linear equations algebraically | Able to apply substitution method correctly and can find the correct solution and can verify it.  
- Able to apply elimination method correctly and can find the correct solution and can verify it.  
- Able to apply cross multiplication method correctly and can find the correct solution and can verify it.  
- Able to identify the most suitable method of solving the pair of linear equations and can find the correct solution and can verify it. | identify the system as consistent or inconsistent  
- Not able to read the exact point of |
| Application in word problems | • Able to identify the variables from given statement  
• Able to frame pair of linear equations correctly  
• Able to solve the equations by any of the above methods  
• Able to verify the solution. | • Not able to identify the variables from the given statement  
• Not able to frame pair of linear equations correctly  
• Not able to solve the equation by any of the above methods  
• Not able to verify the solutions |
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

3. 1 Introduction

You are already familiar with linear equations in one variable and two variables. Recall the equations like $2x + 4 = 0$, $3y – 5 = 0$ etc. are examples of linear equations in one variable and equations like $2x + 5y = 9$ and $y = 3x + 2$ are examples of linear equations in two variables.

In general an equation of the form

$$ax + by + c = 0,$$

where $a$, $b$, $c$ are real numbers and $a$ and $b$ are non-zero is called a linear equation in two variables $x$ and $y$.

(a) $\sqrt{2}x = 1$ is a linear equation in one variable $x$.

(b) $– 3q – 2 = 0$ is a linear equation in one variable $q$.

(c) $0.3 \ell + 0.3 \ m = 2.1$ is a linear equation in two variables $\ell$ and $m$.

Recall, a solution of a linear equation in one variable is a real number which when substituted for the variables makes the equation true.

For example solution of equation $-3q - 2 = 0$ is $q = -\frac{2}{3}$. This value of $q$ on substitution in $-3q - 2$ will equate to zero.
A solution of a linear equation in two variables is a pair of numbers; one for each variable, which when substituted for the respective variables makes the equation true.

For example, $2x + 3y = 7$ has solution $x = 2$ and $y = 1$.

As substitution of these values of $x$ and $y$ in $2x + 3y$ reduce it to $2(2) + 3(1) = 4 + 3 = 7$. So, $x = 2$ and $y = 1$ is a solution of $2x + 3y = 7$.

This solution can also be written as an ordered pair $(2, 1)$ which means $x = 2$, $y = 1$.

**Word of caution:** ordered pair $(2,1) \neq (1,2)$. $(2,1)$ implies that $x = 2$, $y = 1$, while $(1,2)$ implies $x = 1$, $y = 2$.

Observe the graph of equation $2x + 3y = 7$. 

![Graph of equation $2x + 3y = 7$](image)
How many points you can find from graph that satisfies the equation 2x + 3y = 7.

How many solution of this equation are possible?

Is it true for all linear equations?

Note that x = 3, y = \( \frac{1}{3} \) and x = -3, y = \( \frac{13}{3} \) are also solutions of the equation 2x + 3y = 7. You are also aware of graphical representation of a linear equation in two variables. The graph of a linear equation is a straight line such that the ordered pairs representing points on the line are the solutions of the equation and the ordered pairs not on the line are not the solutions of the equation.

As a line has an infinite number of points on it, we can again see that a linear equation in two variables has an infinite number of solutions.

**Point to remember**

A linear equation in two variables has **infinitely many** solutions.

What will be number of solutions if there is a pair of linear equations in two variables say x and y satisfying both the equations?

Will the number of solutions in such a case be still infinite or unique or two or none?

Let us examine all these questions.

**3.2 Pair of Linear Equations in Two Variables**

Two linear equations in the same two variables taken together is said to form a pair of linear equations.

The general form for a pair of linear equations in two variables x and y is

\[
a_1x + b_1y + c_1 = 0
\]
\[ a_2x + b_2y + c_2 = 0 \]

Where \( a_1, b_1, c_1, a_2, b_2, c_2 \) are all real numbers and \( a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0 \).

(i) \[ x + y - 2 + 0 \]
and \[ 3x - 5y + 2 = 0 \]
(ii) \[ 2x + y = 10 \]
and \[ 4x = 3y \]
(iii) \[ 3x + 2y = 12 \]
and \[ 3x + 2y = 18 \]
(iv) \[ 2u + 3v = 5 \]
and \[ 4u + 6v = 10 \]

A **solution** to a pair of linear equations in two variables is an **ordered pair of real numbers** which satisfy both the equations.

For example, \( x = 2 \) and \( y = 1 \) i.e. \((2, 1)\) is a solution of the pair of equation.

\[ 2x + 3y = 7 \]
\[ 3x - 2y = 4 \]

On substituting \( x = 2 \) and \( y = 1 \), in the given equations, we see that left hand side expression becomes equal to the value given on right hand side of the equations:

\[
\begin{align*}
2x + 3y &= 7 \\
3x - 2y &= 4 \\
2 \cdot (2) + 3 \cdot (1) &= 7 \\
3 \cdot (2) - 2 \cdot (1) &= 4 \\
4 + 3 &= 7 \\
6 - 2 &= 4 \\
7 &= 7 \quad \text{(True)} \\
4 &= 4 \quad \text{(True)}
\end{align*}
\]

So, we can say that \( x = 2, y = 1 \) satisfy both the equations.

Thus, \( x = 2, y = 1 \) is a solution of the given pair of equations.
**Example 1:**

Check whether the ordered pair \((2, 3)\) is a solution of given pair of linear equations in two variables.

\[
3x - 2y = 0 \\
4x - 3y = 1 
\]

**Solution:**

Substitute 2 for \(x\) and 3 for \(y\) in both the equations, we get

\[
\begin{align*}
3x - 2y &= 0 \\
4x - 3y &= 1 \\
3 \times 2 - 2 \times 3 &= 0 \\
4 \times 2 - 3 \times 3 &= 1 \\
6 - 6 &= 0 \\
8 - 9 &= 1 \\
0 &= 0 \text{ (True)} \\
-1 &= 1 \text{ (False)}
\end{align*}
\]

So, \((2, 3)\) does not satisfy the equation \(4x - 3y = 1\).

Thus, \((2, 3)\) is not a solution of the given pair of linear equations in two variables. So far, we have checked whether a given ordered pair is a solution of a given pair of equations or not. Now our next step will be how to find an ordered pair (solution) satisfying the given equations.

For that purpose, there are many methods—geometric (graphical) and algebraic.

**Think!**

Linear equations always represent a line in coordinate plane. When two lines are drawn on the same plane, what kind of possible relations they can have?

Does solution has any graphical significance?
3. 3 Graphical Representation of A Pair of Linear Equations in two variables

Let us examine the above stated problems by drawing graphs of different pairs of linear equations:

Example 2:

Draw the graphs of equations $4x - 3y = 6$ and $x + 2y = 7$ on the same Cartesian plane.

Note: As learnt earlier, to draw the graph of linear equations follow the following steps:

1. Express one of the variables in terms of the other.
2. Draw the table of different values of $x$ and $y$.
3. Plot the points to get the lines.

Solution: Tables of values for the equations:

\[
\begin{align*}
4x - 3y &= 6 \\
\text{Or} &\quad y = \frac{4x - 6}{3}
\end{align*}
\]

\[
\begin{array}{c|c|c|c}
 x & 0 & 3 & -3 \\
 Y & -2 & 2 & -6 \\
\end{array}
\]

\[
\begin{align*}
x + 2y &= 7 \\
\text{Or} &\quad x = 7 - 2y
\end{align*}
\]

\[
\begin{array}{c|c|c|c}
 x & 7 & 3 & 1 \\
 y & 0 & 2 & 3 \\
\end{array}
\]

plot the points on Cartesian plane to obtain lines $l_1$ and $l_2$ as shown below:
Observe that these lines intersect each other at one point $P$. Every point on line $\ell_1$ gives a solution of equation $4x - 3y = 6$ and every point on line $\ell_2$ gives a solution of $x + 2y = 7$.

The solution of the pair of linear equations is a unique point $(3, 2)$ i.e. $x = 3$, $y = 2$.

Thus, if the two lines representing a pair of linear equations intersect, then the point of intersection is the unique solution of the given pair of linear equations in two variables.
Example 3:

Draw the graph of the pair of linear equations $2x + 3y = 6$ and $4x + 6y = 24$.

Solution:

Tables of values for the equations:

$$2x + 3y = 6$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>3</th>
<th>-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

$$4x + 6y = 24$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>6</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
The line $\ell_1$ represents the graph of $2x + 3y = 6$ and the line $\ell_2$ represents the graph of $4x + 6y = 24$.

Observe that $\ell_1$ and $\ell_2$ are parallel lines i.e. they do not intersect each other. Thus, there is no point of intersection and hence there is no solution of this pair of equations.

Example 4:

Draw the graphs of pair of linear equations $3x - 2y = 5$ and $6x = 4y + 10$.

Solution

Tables of values for the equations:

<table>
<thead>
<tr>
<th>$3x - 2y = 5$</th>
<th>$6x = 4y + 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>1</td>
</tr>
<tr>
<td>$y$</td>
<td>-1</td>
</tr>
</tbody>
</table>
The lines $\ell_1$ and $\ell_2$ representing the equations $3x - 2y = 5$ and $6x = 4y + 10$ are coincident, i.e. the lines intersect at infinitely many points, implying, there are an infinite number of solutions for this pair of linear equations in two variables.

From the examples 2 to 4 we see that graph of a pair of linear equations is a pair of lines which may be intersecting or may be parallel or coincident.
3. 4 Consistent and Inconsistent system of Linear Equations-

Nature of system of linear equations

When a pair of linear equations in two variables has one or more solutions, then the pair of linear equations is said to be consistent otherwise it is said to be inconsistent.

Thus, the systems in examples 2 and 4 are consistent, while pair of linear equations in example 3 is inconsistent. In example 4, there are infinitely many solutions of the given pair of linear equations. Such a system is called dependent. (Why?)

Thus pair of equation in example 4 is consistent as well as dependent.

Relation between coefficient and nature of system of equations

Let us compare the coefficients of same pair of linear equations. Observe the following table.

<table>
<thead>
<tr>
<th>Pair of Lines</th>
<th>$\frac{a_1}{a_2}$</th>
<th>$\frac{b_1}{b_2}$</th>
<th>$\frac{c_1}{c_2}$</th>
<th>Comparing ratios</th>
<th>Graphical Representation</th>
<th>Algebraic interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4x-3y-6=0</td>
<td>$\frac{4}{1}$</td>
<td>$\frac{-3}{2}$</td>
<td>$\frac{-6}{-7}$</td>
<td>$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$</td>
<td>Intersecting Lines</td>
<td>A unique solution</td>
</tr>
<tr>
<td>x + 2y-7=0</td>
<td>$\frac{2}{4}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{-6}{-24} = \frac{1}{4}$</td>
<td>$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$</td>
<td>Parallel lines</td>
</tr>
<tr>
<td>2x+3y-6=0</td>
<td>$\frac{2}{4}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{-6}{-24} = \frac{1}{4}$</td>
<td>$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$</td>
<td>Parallel lines</td>
</tr>
<tr>
<td>4x+6y-24=0</td>
<td>$\frac{3}{6}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{-2}{-4}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$</td>
<td>Coincident lines</td>
</tr>
<tr>
<td>3x-2y-5=0</td>
<td>$\frac{3}{6}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{-2}{-4}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$</td>
<td>Coincident lines</td>
</tr>
<tr>
<td>6x=4y+10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Can we say that for the pair of linear equations given by (examine for more pair of linear equations)
\[ a_1x + b_1y + c_1 = 0 \]
\[ a_2x + b_2y + c_2 = 0 \]

(1) If \( \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \), the lines intersect at a point and the solution is unique. The pair of equations is consistent.

(2) If \( \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \), the lines are parallel and there is no solution of the equations. The pair of equations is inconsistent.

(3) If \( \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \), the lines are coincident and there are infinitely many solutions. In this case the pair of equations is dependent and consistent. In case of dependent equations, one equation can be obtained from the other by multiplying or dividing the equation by a non zero real number. For example, in example 4, second equation can be obtained from the first by multiplying it by 2.

We can summarize the above observations as follows:

<table>
<thead>
<tr>
<th>Comparing ratios</th>
<th>Graphical Representation</th>
<th>Algebraic interpretation</th>
<th>Nature of system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{a_1}{a_2} \neq \frac{b_1}{b_2} )</td>
<td>Intersecting Lines</td>
<td>A unique solution</td>
<td>Consistent Independent</td>
</tr>
<tr>
<td>( \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} )</td>
<td>Parallel lines</td>
<td>No solution</td>
<td>Inconsistent Independent</td>
</tr>
<tr>
<td>( \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} )</td>
<td>Coincident lines</td>
<td>Infinite solutions</td>
<td>Consistent Dependent</td>
</tr>
</tbody>
</table>
Example 5:
For what value of ‘p’ the following pair of equations has a unique solution.
2x + py = -5
3x + 3y = -6

Solution:
We know that a pair of linear equations has a unique solution
If \( \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \)
Here \( a_1 = 2, \ b_1 = p \)
\( a_2 = 3, \ b_2 = 3 \)
Since, \( \frac{2}{3} \neq \frac{p}{3} \) for unique solution
So \( p \neq 2. \)

For all real numbers except 2, the given pair of linear equations will have a unique solution.

Example 6:
Find the value of k for which the pair of equations
2 x – k y + 3 = 0
4x + 6y – 5 = 0
represent parallel lines.

Solution:
Here
\( a_1 = 2, \ b_1 = -k, \ c_1 = 3 \)
\( a_2 = 4, \ b_2 = 6, \ c_2 = -5 \)

The pair of equations represent parallel lines if
\( \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \)
i.e. \( \frac{2}{4} = \frac{-k}{6} \neq \frac{3}{-5} \) which implies
\( \frac{2}{4} = \frac{-k}{6} \) and \( \frac{-k}{6} \neq \frac{3}{-5} \)
For \( k = -3 \) the given pair of linear equations will represent parallel lines.

**Example 7:**
For what value of \( k \), the pair of equations \( 3x + 4y + 2 = 0 \) and \( 9x + 12y + k = 0 \) represent **coincident** lines.

**Solution**
Here,
\[
\begin{align*}
 a_1 &= 3, & b_1 &= 4, & c_1 &= 2 \\
 a_2 &= 9, & b_2 &= 12, & c_2 &= k
\end{align*}
\]

The pair of equations represent coincident lines if
\[
\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}
\]
\[
\frac{3}{9} = \frac{4}{12} = \frac{2}{k}
\]

i.e. \( \frac{1}{3} = \frac{2}{k} \) or \( k = 6 \)

For \( k = 6 \) the given pair of linear equations will represent coinciding lines.

**Example 8:**
For what value of \( p \) will the following pair of linear equations have **infinitely many** solutions?
\[
\begin{align*}
 P x + 3y - (p - 3) &= 0 \\
 12x + p y - p &= 0
\end{align*}
\]

**Solution:**
Here
\[
\begin{align*}
 a_1 &= p, & b_1 &= 3, & c_1 &= (p - 3) \\
 a_2 &= 12, & b_2 &= p, & c_2 &= -p
\end{align*}
\]

A pair of linear equations has infinitely many solutions if
\[ \frac{a_1}{b_1} = \frac{c_1}{c_2} \]
\[ \frac{p}{12} = \frac{3}{p - (p-3)} \]
\[ \frac{p}{12} = \frac{3}{p} \quad \text{or} \quad \frac{3}{p} = -(p-3), \quad p \neq 0 \]
\[ p^2 = 36 \quad \text{or} \quad 3 = p - 3, \quad p \neq 0 \]
\[ p = \pm 6 \quad \text{or} \quad p = 6 \]

So, required value of \( p \) is 6 as \( p = 6 \) satisfy both the given conditions.

For \( p = 6 \) given pair of linear equations have infinitely many solutions.

### 3.5 Algebraic method of solving pair of linear equations in two variables

The graphical method of solving pair of linear equations provides a quick visualization; of solution(s) but it has its limitations also. Sometimes, it may be difficult to read the exact values of the coordinates of the point of intersection from the graphs particularly when coordinates do not involve integral values but rational values like say \( \left( \frac{7}{3}, \frac{5}{11} \right) \) or irrational values like \( (\sqrt{2}, \sqrt{3}) \) etc.

To overcome this difficulty, we find the solution(s) of the equations by using algebraic methods as explained below:

**Substitution Method**

This method is useful for solving a pair of linear equations in two variables where one variable can easily be written in terms of the other variable.
**Algorithm of substitution method:**

Step 1: Take one of the equations and express one variable say $y$ in terms of $x$.
Step 2: Substitute the value of $y$ obtained in step 1 in the other equation.
Step 3: Simplify the equation obtained in step 2 and find the value of $x$ by solving this equation.
Step 4: Substitute the value of $x$ obtained in step 3 into either equation and solve for second variable.
Step 5: Check the obtained solution by substituting the values of $x$ and $y$ in both the original equations.

**Example 9:** solve the following pair of linear equations by substitution

\[
2x + y = 5 \quad \text{(i)} \\
3x + 2y = 8 \quad \text{(ii)}
\]

**METHOD**

Step 1: Take one of these equations and express one variable say $y$ in terms of $x$.

\[y = 5 - 2x \quad \text{from (i)}\]

Step 2: Substitute the value of $y$ obtained in step 1 in the other equation

Substitute $5 - 2x$ for $y$ in equation (ii)

\[3x + 2y = 8\]

\[3x + 2(5 - 2x) = 8\]

Step 3: Simplify the equation obtained in step 2 and find the value of $x$ by solving this equation.

\[3x + 10 - 4x = 8\]
\[-x = 8 - 10\]
\[-x = -2\]
\[x = 2\]

Step 4: Substitute the value of $x$ obtained in step 3 into either equation and solve for second variable.

Substitute $2$ for $x$ in equation (i)
\[
2x + y = 5 \\
2(2) + y = 5 \\
y = 5 - 4 = 1 \\
\]

Therefore, \( x = 2, y = 1 \) is a solution of the given pair of linear equations in two variables.

**Step 5:** Check the obtained solution by substituting the values of \( x \) and \( y \) in both the original equations.

Check: For (i) equation
L.H.S. = \( 2x + y = 2(2) + 1 = 4 + 1 = 5 \) = R.H.S.

for (ii) equation
L.H.S. = \( 3x + 2y = 3(2) + 2(1) = 6 + 2 = 8 \) = R.H.S.

**Example 10:** Solve the pair of equations
\[
3x + 2y = 12 \quad \text{(i)} \\
4x - y = 5 \quad \text{(ii)}
\]

**Solution:** From equation (ii), we have
\[
y = 4x - 5
\]

Substituting the value of \( y \) in equation (i), we get
\[
3x + 2(4x - 5) = 12 \\
3x + 8x - 10 = 12 \\
11x = 22 \\
x = 2
\]

Substituting the value of \( x \) in equation (ii), we get
\[
4 \times 2 - y = 5 \\
y = 8 - 5 = 3
\]

So, the solution of the given equations is \( x = 2, \ y = 3 \)

**Check:**

Check: For equation (i)
L.H.S. = \( 3x + 2y = 3(2) + 2(3) = 6 + 6 = 12 \) = R.H.S.

for equation (ii)
L.H.S. = \( 4x - y = 4(2) - 3 = 8 - 3 = 5 \) = R.H.S
**Example 11:** Using substitution method, solve the given pair of equations

\[ x + 2y = 4 \quad (i) \]
\[ 2x + 3y = 5 \quad (ii) \]

**Solution:** The given equations are

\[ x + 2y = 4 \quad (i) \]
\[ 2x + 3y = 5 \quad (ii) \]

From equation (i), we have

\[ x = 4 - 2y \]

Substituting this value of \( x \) in equation (ii), we get

\[ 2(4 - 2y) + 3y = 5 \]
\[ 8 - 4y + 3y = 5 \]
\[ y = 3 \]

Substituting the value of \( y \) in equation (i), we get

\[ x + 2x(3) = 4 \]
\[ x = 4 - 6 \]
\[ x = -2 \]

Therefore, the solution is \( x = -2, y = 3 \)

**Check:**

Check: For equation (i),
\[ \text{L.H.S.} = x + 2y = -2 + 2(3) = -2 + 6 = 4 = \text{R.H.S.} \]

For equation (ii)
\[ \text{L.H.S.} = 2x + 3y = 2(-2) + 3(3) = -4 + 9 = 5 = \text{R.H.S.} \]

**Example 12:** Solve the given pair of linear equations, using the substitution method.

\[ x - 5y = 7 \quad (i) \]
\[ 2x - 10y = -5 \quad (ii) \]

**Solution:** From equation (i), we get

\[ x = 7 + 5y \]

Substituting this value of \( x \) in equation (ii), we get

\[ 2(7 + 5y) - 10y = -5 \]
\[ 14 + 10y - 10y = -5 \]
\[ 14 = -5 \quad \text{(which is not possible)} \]

This is a false statement.
So, the pair of linear equations has no solutions.

Note: observe that
\[
\left( \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \right)
\]

**Example 13:** Solve the given pair of linear equations

(i) \[2x + 3y = 6\]
(ii) \[6x + 9y = 18\]

**Solution:** From equation (i), we have
\[
x = \frac{6 - 3y}{2}
\]
or,
\[
x = 3 - \frac{3}{2}y
\]
Substituting this value of \(x\) in equation (ii), we get
\[
6\left(3 - \frac{3}{2}y\right) + 9y = 18
\]
\[
18 - 9y + 9y = 18
\]
\[
18 = 18
\]
This is a true statement but contains no variables. The pair of linear equations is dependent with infinitely many solutions.

Observe that
\[
\left( \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \right)
\]

**Note:** The method of substitution is also known as method of elimination by substitution.

**Elimination Method**

In this method, we eliminate one of the two variables by equating the co-efficient to obtain a single equation in one variable which gives the value of one variable. We explain this method through an example.
Algorithm of elimination method:

**Step 1:** Write both equations in the form $ax + by = c$

**Step 2:** Make the coefficients of one of the variables say $x$, numerically equal by multiplying the equations by suitable real number.

**Step 3:** Add or subtract the equations obtained in step 2 to get an equation in only one variable.

**Step 4:** Solve the equation for the variable obtained.

**Step 5:** Substitute the value of this variable in either of the given equations and find the value of the other variable.

**Step 6:** Check the solution for $(x, y)$ by substituting it in the original equation.

Note: While solving the equation, if we obtain a true statement in step 3 above, the system of equations has infinitely many solutions and if we obtain a false statement, the system has no solutions.

You can check these facts by verifying $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ respectively.

**Example 14:** Solve the pair of equations

\begin{align*}
2x + 5y + 5 &= 0 \quad (i) \\
2y &= 3x + 17 \quad (ii)
\end{align*}

**Solution:**

**Step 1:** Write both equations in the form $ax + by = c$

Rewrite the equations as

\begin{align*}
2x + 5y &= -5 \quad (i) \\
-3x + 2y &= 17 \quad (ii)
\end{align*}
**Step 2:** Make the coefficients of one of the variables say $x$, numerically equal by multiplying the equations by suitable real number.

L.C.M. of coefficients (2 and 3) of $x$ is 6. We multiply equation (i) by 3 and equation (ii) by 2

\[ 6x + 15y = -15 \quad (iii) \]
\[ -6x + 4y = 34 \quad (iv) \]

**Step 3:** Add or subtract the equations obtained in step 2 to get an equation in only one variable.

**Adding equations (iii) and (iv) , we get**

\[ 19y = 19 \]

**Step 4:** Solve the equation for the variable obtained.

\[ y = \frac{19}{19} = 1 \]

**Step 5:** Substitute the value of this variable in either of the given equations and find the value of the other variable.

Substitute 1 for $y$ in equation (i)

\[ 2x + 5y = -5 \]
\[ 2x + 5(1) = -5 \]
\[ 2x = -10 \]
\[ x = -5 \]

The solution is $x = -5, \ y = 1$

**Step 6:** Check the solution for $(x, y)$ by substituting it in the original equation.

**Note:** While solving the equation, if we obtain a true statement in step 3 above, the system of equations has infinitely many solutions and if we obtain a false statement, the system has no solutions.
You can check these facts by verifying respectively.

**Example 15:** Solve the pair of equations

\[
\begin{align*}
x + y &= 3 \\
2x + 5y &= 12
\end{align*}
\]

**Solution:** To eliminate \( x \), we multiply equation (i) by 2 and get

\[
2x + 2y = 6
\]

Subtracting equation (iii) from equation (ii), we get

\[
\begin{align*}
2x + 5y &= 12 \\
2x + 2y &= 6 \\
\hline
3y &= 6 \\
y &= 2
\end{align*}
\]

Putting the value of \( y \) in equation (i), we get

\[
\begin{align*}
x + 2 &= 3 \\
x &= 3 - 2 = 1
\end{align*}
\]

So, solution of the equations is \( x = 1, \ y = 2 \).

Alternatively, to eliminate \( y \), we multiply equation (i) by 5 and get

\[
5x + 5y = 15
\]

Subtracting equation (iii) from equation (ii), we get

\[
\begin{align*}
5x + 5y &= 15 \\
2x + 5y &= 12 \\
\hline
3x &= 3 \\
x &= 1
\end{align*}
\]

Putting the value of \( x \) in equation (i), we get

\[
\begin{align*}
1 + y &= 3 \\
y &= 2
\end{align*}
\]

So, solution of the equation is \( x = 1, \ y = 2 \).
Example 16: Solve the pair of equations

\[ \begin{align*}
31x + 43y &= 117 \\
43x + 31y &= 105
\end{align*} \] (i) (ii)

Solution: We are given

\[ \begin{align*}
31x + 43y &= 117 \\
43x + 31y &= 105
\end{align*} \] (i) (ii)

If we multiply equation (i) by 43 and equation (ii) by 31, then calculation becomes tedious and time consuming because of large numbers in the products.

To avoid this, we adopt the following procedure:

Adding equation (i) and (ii), we get

\[ 74x + 74y = 222 \]

Or \[ x + y = 3 \] (iii)

Subtracting equation (i) from equation (ii), we get

\[ \begin{align*}
12x - 12y &= -12 \\
x - y &= -1
\end{align*} \] (iv)

Again, adding equation (iii) and (iv), we get

\[ 2x = 2 \]

\[ x = 1 \]

Substituting \( x = 1 \) in (iii), we get

\[ 1 + y = 3 \]

Or \[ y = 2 \]

So, solution of the equations is \( x = 1, y = 2 \)

Check:
For equation (i)
\[ \text{L.H.S.} = 31x + 43y = 31(1) + 43(2) = 31 + 86 = 117 = \text{R.H.S.} \]

For equation (ii)
\[ \text{L.H.S.} = 43x + 31y = 43(1) + 31(2) = 43 + 62 = 105 = \text{R.H.S.} \]

Note: The procedure is possible when co-efficient of \( x \) in one equation is the same as co-efficient of \( y \) in other equation and vice-versa.
Cross Multiplication method

Algorithm of cross multiplication method:
To solve the pair of equations
\[ a_1 x + b_1 y + c_1 = 0 \]
\[ a_2 x + b_2 y + c_2 = 0 \]

**Step 1:** Write the co-efficients of \( x \) and \( y \) and the constant terms in the following manner.

\[
\begin{array}{ccc}
\hline
\text{x} & \text{y} & 1 \\
\hline
b_1 & c_1 & a_1 \\
b_2 & c_2 & a_2 \\
\hline
\end{array}
\]

i.e. write co-efficient of \( y \) and constant terms below \( x \).
write constant terms and coefficients of \( x \) below \( y \).
write coefficients of \( x \) and \( y \) below 1.

\[
\begin{array}{ccc}
\hline
\text{x} & \text{y} & 1 \\
\hline
5 & 2 & 4 \\
4 & 1 & 3 \\
\hline
\end{array}
\]

**Step 2:** Multiply numbers written at each arrow, positive sign with arrow pointing downward and negative sign with arrow pointing upward.

\[
\begin{align*}
\frac{x}{b_1 c_2 - b_2 c_1} &= \frac{y}{c_1 a_2 - c_2 a_1} &= \frac{1}{a_1 b_2 - a_2 b_1} \\
\end{align*}
\]

**Step 3:** Taking

\[
\begin{align*}
\frac{x}{b_1 c_2 - b_2 c_1} &= \frac{1}{a_1 b_2 - a_2 b_1} \\
\frac{y}{c_1 a_2 - c_2 a_1} &= \frac{1}{a_1 b_2 - a_2 b_1} \\
\end{align*}
\]

Clearly if, \( a_1 b_2 - a_2 b_1 \neq 0 \), then only we can find \( x \) and \( y \) i.e. Unique solution.
Note: If \( a_1b_2 - a_2b_1 = 0 \), then this method is not applicable. Then the system may be having either no solution or infinitely many solutions. One can find actual status of the equations by comparing the constants.

**Example 17:** Solve the following pair of linear equations by cross multiplication method:

\[
\begin{align*}
2x + 5y &= 17 \\
5x + 3y &= 14
\end{align*}
\]

**Solution:** We can rewrite the given equations as

\[
\begin{align*}
2x + 5y - 17 &= 0 \quad (i) \\
5x + 3y - 14 &= 0 \quad (ii)
\end{align*}
\]

Here \( a_1 = 2, b_1 = 5 \)
\( a_2 = 5, b_2 = 3 \)
\( a_1b_2 - a_2b_1 = 2(3) - 5(5) = 6 - 25 = -19 \neq 0 \)

So, the system has the unique solution.

By cross – multiplication method:

\[
\frac{x}{5} \times (-17) = \frac{y}{-14} \times 2 = \frac{1}{5} \times 3
\]

\[
x = \frac{-70 + 51}{-70 + 51} = \frac{y}{-85 + 28} = \frac{1}{6 - 25}
\]

\[
x = \frac{-19}{-19} = \frac{1}{-19} \quad \text{and} \quad y = \frac{-57}{-57} = \frac{1}{-19}
\]

Or \( x = 1 \) and \( y = \frac{-57}{-19} = \frac{57}{19} = 3 \)

Check: You are advised to check the solution for \( x = 1, y = 3 \).
Example 18: Solve the following pair of linear equations

\[3x + 2y = 13\]
\[6x + 4y = 10\]

Solution: The given pair of linear equations can be written as

\[3x + 2y - 13 = 0\]  \hspace{1cm} (i)
\[6x + 4y - 10 = 0\]  \hspace{1cm} (ii)

Here \(a_1 = 3, b_1 = 2, a_2 = 6, b_2 = 4\)

\[a_1b_2 - a_2b_1 = 3\times 4 - 6\times 2 = 12 - 12 = 0\]

Since, \(a_1b_2 - a_2b_1 = 0\), we cannot apply cross multiplication method in solving these equations.

Check that

\[\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{13}{10}\]

\[i.e. \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}\]

So, the system has no solution.

Applications of linear equations in real life situations

We can solve various daily life problems by converting them into a pair of linear equations in two variables and then solving the pair of equations.

Example 19:

Sum of two numbers is 42 and their difference is 12. Find the numbers.

Solution:

Let the numbers be \(x\) and \(y\)

Sum of the numbers is 42 (given)

So, \(x + y = 42\)  \hspace{1cm} (i)

Difference of the numbers is 12

So, \(x - y = 12\)  \hspace{1cm} (ii)

or \(y - x = 12\)  \hspace{1cm} (iii)
Adding (i) and (ii), we get
2x=54
x=54/2=27
Substituting in equation (i)
27+y=42
y=42-27=15
Solving (i) and (iii) we get
x=15, y=27
Thus, in both case numbers are 15, 27.
Check : Sum of numbers. 15+27=42
Difference of numbers= 27-15=12.

**Example 20:**
The sum of digits of a two digit number is 9. Also nine times this number is twice the number obtained by reversing the order of the number. Find the number.

**Solution:**
Let the unit’s digit be x and the ten’s digit be y.
The required number = 10y+x
On reversing the order of digits
The number = 10x+y
The sum of digits of a two digit number is 9
So, x+y=9 ----------------------(i)
Nine times the number is twice the number obtained by reversing the order of the number, so,
9(10y+x)=2(10x+y)
Or 90y+9x=20x+2y
Or \(-11x+88y=0\)

Or \(x-8y=0\) \(\ldots\) (ii)

Subtracting (ii) from (i)
\[
\begin{align*}
x + y &= 9 \\
x - 8y &= 0
\end{align*}
\]
\[
\begin{array}{c}
\hline
\hline
9y &= 9 \\
y &= 1
\hline
\end{array}
\]

Substituting the value of \(y\) in equation (i)
\[
\begin{align*}
x+1 &= 9 \\
x &= 9-1=8
\end{align*}
\]

Thus, the required number \(=10y+x\)

\[
=10(1)+8=18
\]

Check

1+8=9, sum of digit is 9

\(9(18)=162=2(81)\) which is true

Example 21:

The present age of father is four times the age of his son. Four years later, he will be three times the age of his son. Find their present ages.

Solution:

Let the presents age of father be \(x\) years

And that of son be \(y\) years

Present age of father is four times the age of his son

So, \(x=4y\) \(\ldots\) (i)

Four years later, father’s age will be \((x+4)\) years and son’s age will be \((y+4)\) years
According the question
\((x+4)=3(y+4)\)
Or \(x+4=3y+12\)
\(x=3y+8 \) \(\text{----------(ii)}\)
From equation (i) and equation (ii)
\(4y=3y+8\)
Or \(y=8\)
Substituting the value of \(y\) in equation (i), we get
\(x = 4\times 8\)
\(x = 32\)
Thus, the father’s age is 32 years and the son’s age is 8 years

**Check:** \(32=4(8)\)
Also \(32+4=36\)
\(8+4=12\)
So, \(3(12)=36\)

**Example 22:**
The present age of a father exceeds the sum of the ages of his three children by 8 years. After ten years his age will become \(\frac{5}{6}\) of the sum of ages of the children. Find the present age of the father.

**Solution**
Let the present age of father = \(x\) years and
let the sum of ages of three children = \(y\) years.
After 10 years, father’s age = \((x + 10)\) years.
The Sum of ages of 3 children = \((y+3\times10)\) (why?)
\(= (y + 30)\) years.
According to the problem,

\[ x = y + 8 \]  \hspace{1cm} (i)

Also,

\[ x + 10 = \frac{5}{6} (y + 30) \]

\[ 6x + 60 = 5y + 150 \]  \hspace{1cm} (ii)

Substituting value of \( x \) from equation (i) into equation (ii)

\[ 6 (y + 8) + 60 = 5y + 150 \]
\[ 6y + 48 + 60 = 5y + 150 \]
\[ 6y - 5y = 150 - 108 \]
\[ y = 42 \]

Substituting the value of \( y \) in equation (i)

\[ x = 42 + 8 = 50 \]

So, the present age of father = 50 years
(You can check the solution)

**Example 23:**

In \( \triangle ABC \), \( \angle C = 3 \angle A \) and \( \angle B = 2 (\angle A + \angle C) \) the angles of the triangle.

**Solution**

Let \( \angle A = x \), \( \angle C = y \)

\[ \angle C = 3 \angle A \]
\[ y = 3x \] \hspace{1cm} (1)

Also \( \angle B = 2 (\angle A + \angle C) \)
\[ = 2 (x + y) \]

By angle sum property

\[ \angle A + \angle B + \angle C = 180^\circ \]
\[ x + 2 (x + y) + y = 180^\circ \]
\[ 3x + 3y = 180^\circ \]
or \[ x + y = 60 \] \[ \text{(2)} \]

Substituting value of \( y \) from equation (1) in equation (2)

\[ x + (3x) = 60^\circ \]
\[ x + 3x = 60^\circ \]
\[ 4x = 60^\circ \]
\[ x = \frac{60^\circ}{4} = 15^\circ \]

Substitution value of \( x \) in equation (1)

\[ y = 3 \times 15^\circ = 45^\circ \]

So, \( \angle A = 15^\circ \), \( \angle C = 45^\circ \)

\[ \angle B = 2 (\angle A + \angle C) = 2 (15^\circ + 45^\circ) = 2 \times 60^\circ = 120^\circ \]

**Example 24:**

The area of a rectangle gets reduced by 80 sq units if its length is reduced by 5 units and the breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units the area is increased by 50 sq. units. Find the length and breadth of the rectangle.

**Solution:**

Let the length of rectangle = \( x \) units

breadth of rectangle = \( y \) units

Area of the rectangle = \( xy \) sq. units

According to 1\(^{st}\) condition

\[ (x - 5) (y + 2) = xy - 80 \]

\[ xy = 2x + 5y - 10 = xy - 80 \]

\[ 2x - 5y = -70 \] \[ \text{(i)} \]

According to second condition,

\[ (x + 10) (y - 5) = xy + 50 \]
\[ xy - 5x + 10y - 50 = xy + 50 \]
\[-5x + 10y = 100 \]
or \[ x - 2y = -20 \] \[(ii)\]
or \[ x = 2y - 20 \]
Substituting \(2y - 20\) for \(x\) in equation \((i)\)
\[ 2(2y - 20) - 5y = -70 \]
\[ 4y - 40 - 5y = -70 \]
\[-y = -30 \]
y = 30
Substitution the value of \(y\) in equation \((ii)\)
\[ x - 2(30) = -20 \]
x = -20 + 60 = 40
Thus, the length of the rectangle = 40 units
The breadth of the rectangle = 30 units

**WORK RATIO PROBLEM**

**Example 25**

8 men and 12 women can finish a piece of work in 10 days while 6 men and 8 women can finish it in 14 days. Find the time taken by one man alone to finish the work. Also, find the time taken by one woman to finish the work.

**Solution:**

Let the time taken by one man alone to finish the work = \(x\) days

\[ \therefore \text{One day work of one man} = \frac{1}{x} \]

Let the time taken by one woman alone to finish the work = \(y\) days

\[ \therefore \text{One day work of one woman} = \frac{1}{y} \]
\[ \therefore \text{1 day work of 8 man} = \frac{8}{x} \]

\[ \therefore \text{1 day work of 12 woman} = \frac{12}{y} \]

8 men & 12 woman can finish the work in 10 days

\[ \therefore 10 \left( \frac{8}{x} + \frac{12}{y} \right) = 1 \]

\[ \left( \frac{80}{x} + \frac{120}{y} \right) = 1 \hspace{1cm} \text{(i)} \]

6 men & 12 women can finish the work in 14 days

\[ 14 \left( \frac{6}{x} + \frac{8}{y} \right) = 1 \]

\[ \left( \frac{84}{x} + \frac{112}{y} \right) = 1 \hspace{1cm} \text{(ii)} \]

Substituting \( \frac{1}{x} = u \) and \( \frac{1}{y} = v \) in equation (i) and (ii), we get

\[ 80u + 120v = 1 \]

\[ 84u + 112v = 1 \]

Solving using cross multiplication method

\[
\begin{align*}
\frac{u}{120 - 112} &= \frac{v}{84 - 80} = \frac{-1}{80 \times 112 - 82 \times 120} \\
\frac{u}{8} &= \frac{v}{4} = \frac{-1}{-1120} \\
u &= \frac{8}{1120} = \frac{1}{140} \quad \text{and} \quad v = \frac{4}{1120} = \frac{1}{280}
\end{align*}
\]

\[ x = 140 \quad \text{and} \quad y = 280 \]

One man alone can finish the work in 140 days

One woman alone can finish the work in 280 days

\[ \therefore \text{You can use any method to solve these equations.} \]
Example 26:

Point A and B are 100km apart on a highway. One car starts from A and another from B at the same time. If the cars travel at a constant speeds in the same direction they meet in a hour. Find the speed of the two cars.

Solution:

Let the speed of car starting from point A=x Km/h
Let the speed of car starting from point B=y Km/h
Distance between A and B = 100km
Let the meet at P
When they travel in the opposite directions,
Distance travelled by car station from A=1×x = x
Distance travelled by car station from B=1×y = y
\[ x+y=100 \] (i)
When they travel in the same direction. Let they meet at Q
Distance travel by 1st car A in 5 hour =5x =AQ
Distance travel by 2nd car =5y=BQ
since AQ –BQ=100
so, \[ 5x+5y=100 \]
\[ or \quad x+y=20 \] (ii)
solving equation (i) and (ii) by adding, we get

\[ 2x = 120 \]
\[ x = 60 \]

Putting \( x = 60 \) in (i), we get

\[ 60 + y = 100 \]
\[ y = 40 \]

So, the speed of car starting from point A = 60Km/h

The speed of car starting from point B = 40Km/h

**Example 27:**

A person can row down 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water and speed of the stream.

**Solution:**

Let the speed of rowing in still water = \( x \) km/h

And speed of stream = \( y \) km/h

Speed downstream = \( (x + y) \) km/h

Speed upstream = \( (x - y) \) km/h

He can row down 20 km in 2 hours

\[ \frac{20}{x + y} = 2 \]
\[ 2(x + y) = 20 \]
\[ x + y = 10 \] \( \text{(i)} \)

He can row upstream 4 km in 2 hours

So,

\[ \frac{4}{x - y} = 2 \]
\[ 2(x - y) = 4 \]
\[ x - y = 2 \] \( \text{(ii)} \)
adding equation (i) and (ii)

\[2x=12\]
\[x=6\]

Substituting the value of \(x\) in equation (i), we get

\[6+y=10\]
\[y=4\]

Thus, the speed of rowing in still water = 6 km/h and speed of the stream = 4 km/h (Check the solution with the original question)
STUDENTS’ SUPPORT MATERIAL
Name of the student: ____________________ Date: __________

In which quadrant the given ordered pairs will lie?

- (0, 3)
- (2, -3)
- (0, 2)
- (5, 7)
- (4, 0)
- (-2, 3)
- (-5, -7)
- (2, -3)
- (0, 0)
### Self Assessment Rubric 1 – Warm Up (W1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Understanding</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location of a point in four quadrants</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Location of a point on the x-axis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Location of a point on the y-axis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Location of the Origin.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Student’s Worksheet- 2

Warm up – (W2)

Name of the student: ____________________ Date: ____________

Tell which of the given linear equations are in one variable and which are in two variables? Name the variables also. Read the coefficients of the variables.

(i) \[2x - y = -5\]
(ii) \[-\frac{2}{3}y - z = 6\]
(iii) \[\sqrt{5} - 4x = -7z\]
(iv) \[3x + 8 = 9x\]
(v) \[7y - 3y = 0\]
(vi) \[-5x + 3y = -4x - 2y\]
(vii) \[4x - 3y\]
(viii) \[3x = y\]
(ix) \[3x = 0\]
(x) \[4y\]
(xi) \[5y - 3x = 0\]
### Self Assessment Rubric 2 – Warm Up (W2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Understanding</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can recognize linear equation in one variable.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Can identify a linear equation in two variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Plot the given ordered pairs on the same Cartesian plane.

A. (-1, 3)
B. (2, 5)
C. (3, -2)
D. (-4, -1)
E. (3, 0)
F. (0, 2)
2. Determine the coordinates of each of the points shown in the figure.

<table>
<thead>
<tr>
<th>Points</th>
<th>Coordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td></td>
</tr>
</tbody>
</table>
### Self Assessment Rubric 3 – Warm up (W3)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Green</th>
<th>Light Green</th>
<th>Yellow</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to plot/locate an ordered pair in all four quadrants</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to plot/locate a point on the x-axis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to plot/locate a point on the y-axis</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Parameter Definitions:
- **No Understanding**
- **Understanding of concept but not able to apply**
- **Understanding of concept, can apply but commit errors in calculation**
- **Understanding of concept, can apply accurately**
Name of the student: ___________________ Date: ____________

Read the given instructions and write your answer

Write a linear equation in two variables

1. With coefficient of x less than 3 and coefficient of y more than 5

2. With coefficient of x less -3 and coefficient of y less than 5

3. With coefficient of x equal to 8 and coefficient of y more than 8

4. With coefficient of x and coefficient of y more than 5

5. With sum of the coefficients of a and y as 2 and constant term 3
## Self Assessment Rubric 4 – Pre Content (P1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Understanding</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has knowledge of terms coefficients, constant term, variable</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to write a linear equation in two variable with given coefficients</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Fill in the abscissa or the ordinate so that the given ordered pair is a solution of the indicated equation.

   (1) $x + 3y = 5$ 
   (2) $2x - 3y = 7$ 
   (3) $3x + 2y = 7$ 
   (4) $x - 2y = 6$

   (1) (-----, -2) 
   (2) (-4, ------) 
   (3) ($\frac{1}{3}$, -------) 
   (4) (0, -------)

2. Complete the ordered pair so that all of them are solutions of $2x + 3y = 6$

   (1) (3,  )
   (2) (0,  )
   (3) (4,  )
   (4) ( , 1)

3. The graph of the equation $x + y = 10$ is a .................

   (a) Curved line
   (b) Straight line
   (c) Straight line Passing through origin
   (d) Straight line parallel to x -axis
### Self Assessment Rubric 5 – Pre Content (P2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to find solution of a given linear equation in two variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to verify that a given point is a solution of linear equation in two variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know that the graph of a linear equation in two variables is a straight line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Plot the coordinates given in the following table and join them.

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>2</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>5</td>
<td>0</td>
<td>10</td>
</tr>
</tbody>
</table>

What do we get?

__________________________________

Is it a line or line segment?

__________________________________
Locate two more points on it.

________________________________________________________________________

How many more points can you locate on the line?

________________________________________________________________________

2. Plot the coordinates given in the following tables on the same Cartesian plane.

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>0</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>-3</td>
<td>-6</td>
</tr>
</tbody>
</table>

What type of lines are they?

________________________________________________________________________

Do the lines have any point common?

________________________________________________________________________
If yes, how many?

3. Represent graphically the pair of linear equations in two variables:
   \[ 2x + y = -4 \]
   \[ 4x + 2y = 8 \]

   Complete the tables and draw the graph.

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>-1</th>
<th>0</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What types of line are obtained?

How many points are common in the two lines?

4. Represent graphically the pair of linear equations in two variables:

\[ x - 2y = 2 \]
\[ -2x + 4y = -4 \]
### Self Assessment Rubric 6 – Content (CW1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to plot the given ordered pairs</td>
<td>✔</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to observe the graph as a straight line</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to plot a pair of linear equations in two variables by making table of values</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to represent intersecting, parallel or coincident lines using pair of equations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Given below is the graph of a pair of linear equation in two variables. Observe it and answer the questions given.

What types of lines are obtained?
________________________________

How many points are common in the two lines?
________________________________

Does (3, -1) lie on both the equations? Explain
___________________________________________________________________
2. Represent graphically the given pair of linear equations in two variables:
\[ x - y = 2, \quad x + y = 6 \]

Make a table of values for both the equations:

\[
\begin{array}{c|c|c}
X & 0 & 1 \\
Y & 0 & 1 \\
\end{array}
\]

We observe the lines intersect at \------------------

\[ \therefore \text{The pair of equations has a unique/infinite/no solution(s)} \]

Verify that the point of intersection lies on both lines.
3. Draw the graphs of the following pair of equations

\[ 2x + y = 2 \]
\[ 2x + y = 8 \]

Table of values for \( 2x + y = 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table of values for \( 2x + y = 8 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The lines obtained are

The lines have ___________________________ points in common.
The given pair of equation has ________________________________ solution.

5. Solve graphically the linear equations by taking suitable values of x and y.

\[
\begin{align*}
2x + 5y &= 1 \\
4x + 10y &= 2
\end{align*}
\]

\begin{tabular}{|c|c|}
\hline
X & \\
\hline
Y & \\
\hline
\end{tabular}

What do you observe?

_____________________________________________________

The given pair of linear equations has __________ solution(s).
**Brainstorming:**

What types of graphs are possible?

What types of solutions are possible?

---

**Self Assessment Rubric 7 – Content (CW2)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Understanding</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to plot pair of linear equations on graph</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to observe the types of graphs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Name of the student: ___________________________ Date: ____________

1. Given a linear equation in 2 variables, identify the following.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Linear equation</th>
<th>Coefficient of x</th>
<th>Coefficient of y</th>
<th>Constant term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$2x + 3y - 7 = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>$3x - 2y - 7 = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>$4x + 5y + 14 = 0$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Task 1

Investigate the given pairs of equations. Find the ratios of coefficients of $x$, coefficients of $y$ and constant terms. Write your observations. Draw their respective graphs using (GeoGebra) and express your result in terms of conditions for consistency or inconsistency.

\[
a_1x + b_1y = c_1 \\
a_2x + b_2y = c_2
\]

Relation between the observed ratios

1. $11x - 6y = 3$, 
   $22x - 12y = 17$
2. $9x - 5y = 0$, 
   $-27x + 15y = 4$
3. \[ 18x + 13y = 14, \]
   \[ 54x + 39y = 17 \]
4. \[ 5x + 3y = 19, \]
   \[ 20x + 12y = 14 \]
5. \[ 15x + 12y = 4, \]
   \[ 5x + 4y = 6 \]
6. \[ 20x + 20y = 10, \]
   \[ 20x + 20y = 5 \]
7. \[ 19x + 19y = 18, \]
   \[ x + y = 8 \]
8. \[ 6x + 2y = 16, \]
   \[ 6x + 2y = 20 \]
9. \[ 2x + 12y = 18, \]
   \[ 12x + 72y = 20 \]
10. \[ 3x + 11y = 14, \]
   \[ 15x + 55y = 4 \]

3. Task 2

Investigate the given pairs of equations. Find the ratios of coefficients of \( x \),
coefficients of \( y \) and constant terms. Write your observations. Draw their
respective graphs using (GeoGebra) and express your result in terms of conditions
for consistency or inconsistency.

\[ \frac{a_1}{a_2} \quad \frac{b_1}{b_2} \quad \frac{c_1}{c_2} \quad \text{Relation between the observed ratios} \]
1. \(11x - 6y = 3,\)
   \(2x - y = 17\)
2. \(19x - 5y = 0,\)
   \(-27x + 15y = 4\)
3. \(18x + 13y = 14,\)
   \(54x + 39y = 17\)
4. \(15x + 13y = 19,\)
   \(20x + 12y = 14\)
5. \(15x + 12y = 4,\)
   \(25x + 14y = 6\)
6. \(20x + 10y = 10,\)
   \(10x + 20y = 5\)
7. \(19x + 9y = 18,\)
   \(x + 5y = 8\)
8. \(x + y = 16,\)
   \(6x + 2y = 20\)
9. \(3x + 12y = 18,\)
   \(12x + 2y = 20\)
10. \(5x + 11y = 14,\)
    \(15x + 55y = 4\)
4. Task 3

Investigate the given pairs of equations. Find the ratios of coefficients of \(x\), coefficients of \(y\) and constant terms. Write your observations. Draw their respective graphs using (GeoGebra) and express your result in terms of conditions for consistency or inconsistency.

\[
\begin{align*}
a_1x + b_1y &= c_1 \\
a_2x + b_2y &= c_2
\end{align*}
\]

Relation between the observed ratios

1. \(11x - 6y = 3, \quad 22x - 12y = 6\)
2. \(9x - 5y = 1, \quad -27x + 15y = -3\)
3. \(18x + 13y = 14, \quad 54x + 39y = 42\)
4. \(5x + 3y = 19, \quad 20x + 12y = 76\)
5. \(15x + 12y = 6, \quad -5x - 4y = -2\)
6. \(10x + 10y = 5, \quad 20x + 20y = 10\)
7. \(19x + 19y = 19, \quad x + y = 1\)
8. \(6x + 2y = 16, \quad \)
36x + 12y = 96
9. 2x + 12y = 3,
12x + 72y = 18
10. 3x + 11y = 14,
15x + 55y = 70

<table>
<thead>
<tr>
<th>Pair of equations</th>
<th>Graphical representation</th>
<th>( \frac{a_1}{a_2} )</th>
<th>( \frac{b_1}{b_2} )</th>
<th>( \frac{c_1}{c_2} )</th>
<th>Relation between ( \frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>x - y - 2 = 0</td>
<td>x + y = 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x + y - 6 = 0</td>
<td>x - y = 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. \(x + y = 3\)
   \(x - 3y = -3\)

2. \(2x + y - 2 = 0\)
   \(2x + y - 6 = 0\)

3. \(y = 3x + 8\)
   \(y = 3x - 4\)
1. When pair of linear equations represents intersecting lines then
   _______________________________________________________________________

2. When pair of linear equations represents parallel lines, then
   _______________________________________________________________________

3. When pair of linear equations represents coincident lines, then
   _______________________________________________________________________

106
4. How many solutions do two linear equations in two variables have if their graphs?

1. Intersect at one point
   __________________________________________

2. Do not intersect at any point
   __________________________________________

3. Represents coincident lines
   __________________________________________

Self Assessment Rubric 8 – Content (CW3)

Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Understanding</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to write the ratio of coefficients of variables and constant terms</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to find the condition for unique solution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Able to find the condition for many solution

Able to find the condition for no solution

Able to find the condition for intersecting lines, coincident lines and parallel lines

Student’s Worksheet- 9, Relation between Coeffs of x and y

Content Worksheet - (CW4)

Name of the student: ___________________ Date: __________

1. How many solutions of the linear equation 5x -3y = 9 are possible? Justify your answer.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

2. When do you say the given pair of equations is consistent?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

3. When do you say the given pair of equations is inconsistent?

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
4. Determine whether \((3, -7)\) is a solution of the pair of equations. Is the given pair of equations consistent or inconsistent? How do you conclude?

\[
\begin{align*}
  x + y + 4 &= 0 \\
  4x - 3y - 11 &= 0
\end{align*}
\]

5. A pair of linear equations in two variables is called consistent if it has

(a) Unique solution
(b) Infinitely many solutions
(c) No solution
(d) Either (a) or (b)

6. A pair of linear equations in two variables is inconsistent if the graph represents _______________

(a) Intersecting lines
(b) Parallel lines
(c) Consistent lines
(d) Both (a) and (b)

7. The equations \(x + 2y = 4\) and \(2x + y = 5\) are

(a) Inconsistent
(b) Consistent & have a unique solution
(c) Consistent & have infinitely many solutions

8. Write a linear equation in two variables which is consistent to the equation \(3x - 5y + =0\).
9. Fill in the box with a suitable number to make the given pair of equations inconsistent.

\[ 2x + 3y = 3 \]

\[ 4x + 6y = \]

**Self Assessment Rubric 9 – Content (CW4)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Understanding</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to define a consistent system of equations</td>
<td>☐</td>
<td>☑</td>
<td>☐</td>
<td>☑</td>
</tr>
<tr>
<td>Able to define an inconsistent system of equations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to tell the condition for consistency or inconsistency</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Without drawing the graph, by observing the coefficients of the pair of linear equations comment open the nature of the system.

\[ x + y = 3 \]
\[ 2x + 5y = 12 \]

**Ratio of coefficients of \( x \) = \[ \frac{a_1}{a_2} = \] \[ \frac{1}{2} \]

**Ratio of coefficients of \( y \) = \[ \frac{b_1}{b_2} = \] \[ \frac{1}{-\frac{5}{2}} = \frac{-1}{2} \]

What do you observe?

________________________________________________________________________________________

The given pair of equations will be (intersecting/parallel/coincident)

________________________________________________________________________________________

The system is ____________________________________________

(inconsistent/ consistent)

2. Given linear equation in two variables: \( x + y = 6 \)

Form an equation in two variables such that it is inconsistent to the given equation.

________________________________________________________________________________________

What type of lines will they represent on the graph?

________________________________________________________________________________________
3. The following pairs of linear equations represent coincident lines. Replace the ‘?’ symbol in both the equations with suitable constants.
   a) \[3x + ? y = -5\]
      \[?x + 8y = -10\]
   b) \[a + ? b = 3\]
      \[-2a + 4b = ?\]
   c) \[-u + ? v = 6\]
      \[u - 3v = ?\]

   Hence the above pair of linear equations in two variables represents a ________ system.

4. \[2x + 5y = 9\] is a linear equation in two variables.
   \[4x + ? y = 18\] is another linear equation in two variables.

   Replace ‘?’ such that pair of equations
   a) are intersecting: ______________________
   b) are coincident: ______________________
<table>
<thead>
<tr>
<th>Parameter</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Without actually drawing the graph writing consistent pairs of equations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Able to write inconsistent pairs of equations</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. Solve the given pair of linear equations in two variables by substitution method.

   \[ 5x - 3y = -2 \] ................................(i)
   \[ -x + 2y + -8 = \] ........................... (ii)

   From (i) \[ x = \] ........................................ (iii)

   Put the value of \( x \) obtained in terms of \( y \) as in (iii) in any of (i) or (ii) and solve
   \[ \text{________________________________________________________} \]
   \[ \text{________________________________________________________} \]
   \[ \text{________________________________________________________} \]

   Substituting value of \( y \) in (iii), we get.
   \[ x = \]
   \[ = \]
   \[ = \]

   Answer:
   \[ x = , \quad y = \]

   Check the solution:

2. Examine if the given pair of equations has a unique solution, then solve them by substitution method, by completing the boxes:

   \[ 3x + 2y = 6 \] .................................
2x + y = 10
\[ \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \]

\[ \frac{a_1}{a_2} = \frac{b_1}{b_2} \quad : \text{Which type of solution exists?} \]

Now, from equation , Write x in terms of y

x =

Put value of ‘x’ in , and solve for ‘y’

\[ \text{solution is} \]

\[ x = \]
\[ y = \]

Check the solution

3. Consider the given pair of equations:

-2x + y = 4

4x - 2y = 7

Applying substitution method from equation

\[ y = \]

Which is the next step?

What do we get offer solving?
What do you conclude from this?
___________________________________________________

We can verify it by comparing the coefficients of the two equations.
Examine it.
\[
\frac{a_1}{a_2} = \ldots, \quad \frac{b_1}{b_2} = \ldots, \quad \frac{c_1}{c_2} = \ldots
\]

What relation do they have:
___________________________________________________

4. On solving a pair of linear equations by substitution method the end result is of the form \(12 = 12\). What does it imply?

___________________________________________________

Also formulate an example of a pair of linear equations in two variables. Which would fetch the end result of the form \(12 = 12\) ‘on applying substitution method.

Equation ..........................................................
Equation ..........................................................

Solve the following pairs of linear equations in two variables by Substitution method.

1.) \(x = -5y - 53 \quad x + 3y = -35\)
2.) \(y = -2x + 22 - x = 10 - 3y\)
3.) \(x = -3y + 27 x = -4y + 35\)
4.) \(x = -5y - 12 \quad x = -2y + 0\)
5.) \(-5y - 5x = 35 \quad y = -2x - 17\)
6.) \(y = -x - 17 - x = -1 - y\)
7.) \(y + x = 2 \quad y = x + 4\)
8.) \(x = -5y + 50 \quad x - 4y = -40\)
9.) \(y = -5x + 40 \quad 5x = 40 - 3y\)
10.) \(x = y + 2 \quad x + 5y = 44\)
### Self Assessment Rubric 11 – Content (CW6)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No Understanding</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to express one variable in terms of the other</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
<tr>
<td>Able to find the solution (if any) using the substitution method.</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
<td>☐</td>
</tr>
</tbody>
</table>
Student’s Worksheet- 12, Solution of Pair of Linear Equations in Two Variables

Content Worksheet - (CW7)

Name of the student: ___________________________ Date: ____________

1. Given the pair of equations
   \( x + y = 11 \)
   \( x - y = 5 \)
   Which variable is eliminated on adding the two equations?

2. \( x - 2y =3 \)
   \( 3x - 2y = 3 \)
   The variable \( y \) is eliminated on subtracting the equations.

3. Given the pair of pair of equations
   \( 3x + 2y + 1 =0 \) .............................. (1)
   \( 2x -3y + 8 = 0 \) .............................. (2)
   To eliminate \( x \), we multiply
   Equation (1) by ..............................
   and equation (2) by ..............................

4. Given the pair of equations
   \( 14x + 3y - 17 = 0 \) .............................. (1)
   \( 21x -13y - 8 = 0 \) .............................. (2)
   to eliminate \( x \), we multiply equation (1) & (2) by suitable constants so as to get ..........................of 14 and 21.

   Solve using elimination method

5. \( -5+x=-4(y+4); -2x-6y=3(y-8) \)
6. \( -2(x+8) =3+8y; 3-8x=3(1+4y) \)
7. \( -2-7y=-8(1+5x); -5y-6x=-7(x+8) \)
8. \( 6+y=-6(x-3); -4+y=-5(x-8) \)
9. \( -7-4y=-4(1-6x); x=-4(-7-y)-4 \)
### Self Assessment Rubric 12 – Content (CW7)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Understanding of concept but not able to apply</th>
<th>Understanding of concept, can apply but commit errors in calculation</th>
<th>Understanding of concept, can apply accurately</th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to eliminate one variable</td>
<td><img src="#" alt="Green" /></td>
<td><img src="#" alt="Green" /></td>
<td><img src="#" alt="White" /></td>
</tr>
<tr>
<td>Able to solve pair of equations using elimination method</td>
<td><img src="#" alt="Green" /></td>
<td><img src="#" alt="Green" /></td>
<td><img src="#" alt="White" /></td>
</tr>
</tbody>
</table>
Student’s Worksheet- 13, Solution by Substitution Method

Content Worksheet - (CW 8)

Name of the student: ____________________  Date: __________

Find 20 words from the chapter pair of linear equations in two variables. They are placed horizontally, vertically, diagonally as well as backward.

The word search is also available online
Answer Key

linear equation  coordinate axes  pair of equations  consistent system
unique solution  ordered pair  variables  infinite solutions
inconsistent system  no solution  coefficients  graphical method
constant  algebraic method  intersecting  elimination
parallel  substitution  coincident  cross multiplication
1. The sum of two numbers is 35 and their difference is 13. Find the numbers.  
   Identify the two conditions in the given problem.  
   Let the two numbers are .......... and ...........
   The sum of two numbers is 35  
   Express it as a linear equation ........................................
   The difference of numbers is 13  
   Express it as a linear equation ........................................
   D

2. Saumya has pens and pencils which together are 40 in number. If she has  
   five more pencils and five less pens, then number of pencils would become  
   four times the number of pens.  
   If Soumya has x number of pens and y number of pencils,  
   Then x +y =..........................  
   If she has five more pencils, then number of pencils =...................... and  
   five less pens, then number of pens = ........................................  
   We can represent the second condition algebraically as  
                                                                                      

3. 10 years ago father was 12 times as old as his son & 10 years hence he will  
   be twice as old as his son. Find their present ages.  
   Identify the two conditions in given problem  
   Condition 1:  
                                                                                      
   Condition 2:  
                                                                                      
Let father’s age be \( x \) years & son’s age be \( y \) years, then ten years ago,

- Father’s age = \( \ldots \) 
- Son’s age = \( \ldots \)

and ten years hence,

- Father’s age = \( \ldots \) 
- Son’s age = \( \ldots \)

The first condition can be represented algebraically as (write the equation for first condition)

\[
\text{equation for first condition}
\]

Write the equation for the second condition

\[
\text{equation for second condition}
\]

Now solve the two equations to find the present age of father and son.

**Self Assessment Rubric 14 – Content (CW9)**

- No Understanding
- Understanding of concept but not able to apply
- Understanding of concept, can apply but commit errors in calculation
- Understanding of concept, can apply accurately
Student’s Worksheet- 15, Solution by Cross Multiplication Method

Content Worksheet - (CW 10)

Name of the student: ___________________ Date: _________

After reading the given explanation, solve the pair of equations using cross multiplication method.

Let us solve the pair of equations

\[ a_1x + b_1y + c_1 = 0 \]
\[ a_2x + b_2y + c_2 = 0 \]

For example:
\[ 4x + 5y + 2 = 0 \quad (i) \]
\[ 3x + 4y + 1 = 0 \quad (ii) \]

Method:
Step 1: Write the co-efficients of x and y and the constant terms in the following manner.

\[ \frac{x}{b_1} = \frac{y}{c_1} = \frac{1}{a_1} \]
\[ b_2 \quad c_2 \]
\[ a_2 \quad b_2 \]
i.e. write coefficient of $y$ and constant terms below $x$.
write constant terms and coefficients of $x$ below $y$.
write coefficients of $x$ and $y$ below 1.

$$
\begin{array}{ccc}
\text{Step 2: } & \text{Multiply numbers written at each arrow, positive sign with arrow pointing downward and negative sign with arrow pointing upward.} \\
\frac{x}{5} & = & \frac{y}{2} \\
\frac{4}{1} & = & \frac{1}{1} \\
\frac{2}{1} & = & \frac{3}{4} \\
\frac{1}{3} & = & \frac{4}{5}
\end{array}
$$

Step 3: Taking and find values of $x$ and $y$.

$$
\begin{array}{ccc}
\frac{x}{-3} & = & \frac{1}{1} \\
\frac{y}{2} & = & \frac{1}{1} \\
\end{array}
$$

Therefore, the solution is $x = -3, y = 2$

Clearly if, $a_1b_2 - a_2b_1 \neq 0$, then only we can find $x$ and $y$. i.e. unique solution.

Clearly here $a_1b_2 - a_2b_1 = 1 \neq 0$

So, $x = -3, y = 2$ is a unique solution.
Note: If \( a_1b_2 - a_2b_1 = 0 \), then this method is not applicable. Then the system may be having either no solution or infinitely many solutions. You can find actual status of the equations by comparing the constants.

Solve the following pairs of linear equations using the cross multiplication method.

1.) \( x = -5y - 53 \) \( x + 3y = -35 \)
2.) \( y = -2x + 22 \) \( -x = 10 - 3y \)
3.) \( x = -3y + 27 \) \( x = -4y + 35 \)
4.) \( x = -5y - 12 \) \( x = -2y + 0 \)
5.) \( -5y - 5x = 35 \) \( y = -2x - 17 \)
6.) \( y = -x - 17 \) \( -x = -1 - y \)
7.) \( y + x = 2 \) \( y = x + 4 \)
8.) \( x = -5y + 50 \) \( x - 4y = -40 \)
9.) \( y = -5x + 40 \) \( 5x = 40 - 3y \)
10.) \( x = y + 2 \) \( x + 5y = 44 \)

Self Assessment Rubric 15 – Content (CW10)
**Parameter**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Able to solve the pair of linear equations using cross multiplication method.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**Dialogue Dilemma**

After reading the given dialogues, answer the following questions.

- **Gopal**: I have found one solution of the pair of equations $2x + 3y = 7$; $2x + y = 7$
- **Radha**: I am not able to find any solution of pair of equations $2x + 3y = 7$; $6x + 9y = 1$.
- **Jasmine**: I have found one solution of the pair of equations $2x + 3y = 7$; $4x + 6y = 14$.
- **Sahiba**: I have found two solutions of the pair of equations $2x + 3y = 7$; $8x + 12y = 28$.

---

**Name of the student:** _____________________  **Date:** ________
1. Do you think Radha will be able to find any solution to the pair of equations given to her? Justify your answer.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

2. Suggest her new coefficient of x for the second equation so that she can find exactly one solution for the pair of equations given to her.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

3. Do you think Jasmine will be able to find more solutions to the pair of equations given to her? Justify your answer.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

4. Do you think Gopal will be able to find any solution to the pair of equations given to her? Justify your answer.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

5. Do you think Sahiba will be able to find any solution to the pair of equations given to her? Justify your answer.
6. Write your views on solutions obtained by Jasmine and Sahiba.

________________________________________________________
________________________________________________________
________________________________________________________
________________________________________________________
________________________________________________________

Student’s Worksheet- 17
Post Content Worksheet - (PCW2)

Name of the student: ___________________ Date: __________

For the given pairs of equations, without actually drawing the graph, tell whether the pair of lines will intersect, coincide or be parallel?

Name: ___________________

1. $6x + 10y = 20, \ 5x + 19y = 6$

________________________________________________________
________________________________________________________

2. $8x + 9y = 11, \ 16x + 18y = 8$

________________________________________________________
________________________________________________________
3. \(4x + 16y = 16, \ 12x + 48y = 48\)


4. \(17x + 16y = 7, \ 18x + 18y = 18\)


5. \(6x + 5y = 8, \ 12x + 10y = 8\)


6. \(19x + 15y = 9, \ 19x + 9y = 15\)


7. \(2x + 7y = 20, \ 2x + 7y = 16\)


8. \(13x + 13y = 13, \ 13x + 13y = 4\)


9. \(5x + 2y = 5, \ 10x + 20y = 10\)


10. \(8x + 18y = 10, \ 8x + 17y = 5\)


130
What is the general equation for a linear equation in two variables? Write some examples also.

The equations like $2x + 5y = 9$ and $y = 3x + 2$ are examples of linear equations in two variables.

The equations like $2x + 4 = 0$, $3y - 5 = 0$ etc. are examples of linear equations in one variable.
Write a linear equation in two variables. Find its 3 solutions.

______________________________

______________________________

______________________________

______________________________

______________________________

______________________________
Take a linear equation in two variables. Draw its graph. Find its 3 solutions.

____________________________________________________________________

____________________________________________________________________

____________________________________________________________________

____________________________________________________________________

____________________________________________________________________

____________________________________________________________________
Without actually drawing the graph, write 3 pairs of equations for each case.
What is the condition for a pair of equations to have solution(s)?

I think when two ordered pairs satisfy the given pair of linear equations in two variables then there can be infinite solutions.

What do you say?
Student’s Worksheet- 19

Post Content Worksheet - (PCW4)

Name of the student: _____________________  Date: __________

Assignment 1

1. Write 5 linear equations in two variables parallel to \( x+4y-7 =0 \). What condition did you use?
2. Write 5 linear equations in two variables intersecting with \( x+4y-7 =0 \). What condition did you use?
3. Write 5 linear equations in two variables coinciding with \( x+4y-7 =0 \). What condition did you use?
4. If in a given pair of linear equations the ratio of coefficient of \( x \) is not equal to ratio of coefficient of \( y \), what kind of lines will their graph represent?
5. Comment on the type of graph for the given pair of linear equations in two variables.
   a. \( 5x+2y=3; 10+4Y=4 \)
   b. \( 2x+3y=13; 5x+2y=16 \)

6. Which of the following system of equation has a unique solution?
   a. \( 3x+y=2,6x+2y=3 \)
   b. \( x-2y=3,3x-2y =1 \)
   c. \( 2x-5y=3,6x-15y=9 \)
   d. \( 2x+3y=4,4x+6y=8 \)

7. Find the condition for the following system of linear equations to have a unique solution
   a. \( 2X+PY+=0,5X+QY+S=0 \)
   b. \( zx+by+7=0,lx+my+11=0 \)
   c. \( 3a+5b+8=0,ma+nb+8=0 \)

7. Find the value of \( k \) so that the pair of equations represents
   a. Parallel lines
b. Intersecting lines

\[ Kx + 3y = 5, \ 4x + y = 11 \]

8. What is the name given to a system of simultaneous linear equations if it has no solution?

9. What is the minimum number of solutions a pair of linear equations must have if it is a consistent system?

10. What is the maximum number of solutions a pair of linear equations can have if it is a consistent system?

11. What is the nature of graph of a system of linear equations which is inconsistent?

12. If a pair of linear equations is consistent then the lines will be
   (a) Always intersecting (b) parallel (C) intersecting or coincident (d) always coincident

13. What is the condition for pair of linear equations to be inconsistent

   \[ ax + by = c, lx + my = n \]

14. For what value of \( k \), the pair of linear equations is consistent?

   \[ 2x + ky = 1; \ x - 3y = -3 \]

15. One equation of a pair of inconsistent linear equations is \( x - 4y = 8 \) The second equation can be
   (a) \( 2x - 8y = 16 \)
   (b) \(-3x + 12y = 8\)
   (c) \( 3x + 12y = 8\)
   (d) \( x + 4y = 8\)
Student’s Worksheet- 20
Post Content Worksheet - (PCW5)

Name of the student: ___________________ Date: ____________

1. Form a pair of linear equations representing the given situations
   (a) 10 friends went for a picnic & the number of girls is 4 more than the number of boys.
   (b) 5 pencils & 7 pens together cost Rs 45 whereas 7 pencils & 5 pens together cost Rs 39.
2. The sum of the digits of a two digit number is 7. When the digits are reversed, the number is decreased by 9 find the number.
3. Five years hence, fathers age will be three times the age of his son. Five years ago, father was seven times as old as his son. Find their present ages.
4. 8 men & 12 women can finish a piece of work in 10 days while 6 men & women can finish it in 14 days. Find the time taken by one men alone & that by one women alone to finish the work.
Project using ICT

Description:

1. Write 10 pairs of linear equations in two variables forming intersecting lines.

Exploration 1:

2. Deliberately take ratio of coefficients of x equal to ratio of constant terms.
3. Draw the graphs and observe the location of point of intersection.
4. Write your observations.

Exploration 2:

5. Deliberately take ratio of coefficients of x equal to ratio of constant terms.
6. Draw the graphs and observe the location of point of intersection.
7. Write your observations.

Note: You may use open source software GeoGebra

Prepare a project report.
Useful Online Links

2. [http://www.youtube.com/watch?v=cHH_NqNuwYI](http://www.youtube.com/watch?v=cHH_NqNuwYI) for learning to find solution of a linear equation in two variables
3. [http://www.youtube.com/watch?v=VKqledd8wUA](http://www.youtube.com/watch?v=VKqledd8wUA) for plotting points
5. [http://www.youtube.com/watch?v=4h65y9Xj4eY&feature=related](http://www.youtube.com/watch?v=4h65y9Xj4eY&feature=related) for learning to plot linear equation on graph
6. [http://www.youtube.com/watch?v=jpVrLZdluW0](http://www.youtube.com/watch?v=jpVrLZdluW0)
7. [http://www.youtube.com/watch?v=R-4FkXOGDQC](http://www.youtube.com/watch?v=R-4FkXOGDQC) Consistent and inconsistent system
8. [http://www.youtube.com/watch?v=VqWfxtc2vCg&feature=related](http://www.youtube.com/watch?v=VqWfxtc2vCg&feature=related) Meaning of consistent and inconsistent system
10. [http://www.youtube.com/watch?v=mDw2F2zvThs&feature=player_embedded#at=210](http://www.youtube.com/watch?v=mDw2F2zvThs&feature=player_embedded#at=210) Substitution method
12. [http://www.youtube.com/watch?v=ClBb7YR0FHQ&feature=related](http://www.youtube.com/watch?v=ClBb7YR0FHQ&feature=related) Elimination method part 2

Extra reading:

4. http://www.mbs.edu/home/jgans/mecon/value/Popups/pop_up_analytical_methods.htm
5. Graphing linear equations in two variables
   http://www.tpub.com/math1/13.htm
9. Maths activity 1
   http://mykhmsmathclass.blogspot.com/search/label/Activity-Linear%20equations%20in%20two%20variables%20%28I%29
10. Maths activity 2
    http://mykhmsmathclass.blogspot.com/search/label/Activity-Linear%20equations%20in%20two%20variables%20%28II%29
11. Maths activity 3
    http://mykhmsmathclass.blogspot.com/search/label/Activity-Linear%20equations%20in%20two%20variables%20%28III%29
12. Applications of linear equations in two variables exercise
13. Discuss examples substitution method
15. Points to remember